Stabilität und Konvergenz der Euler-Verfahren

Tobias Jahnke

Karlsruher Institut für Technologie

Vorlesung *Quantitative Biologie*

Sommersemester 2012

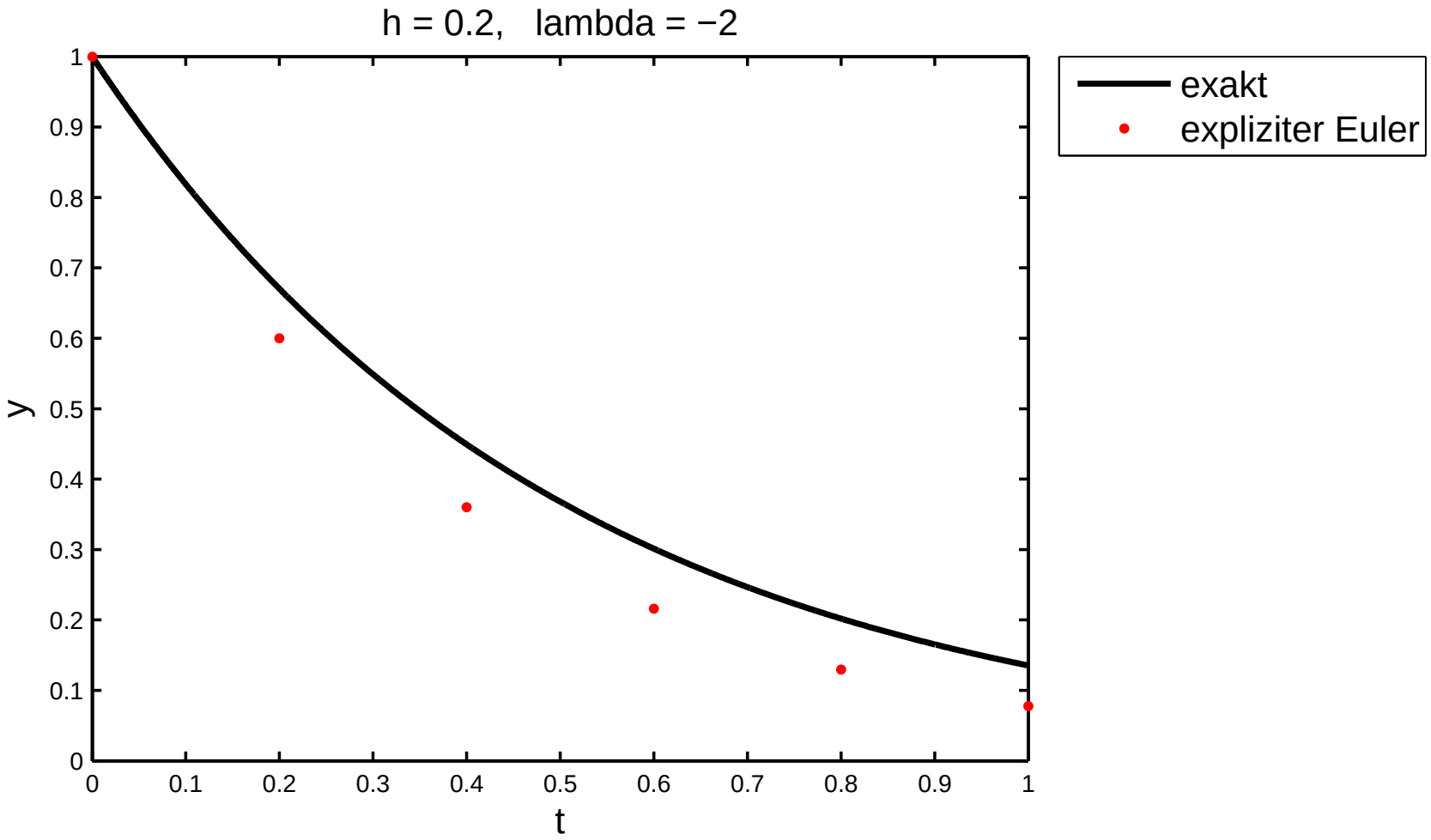
Testproblem:

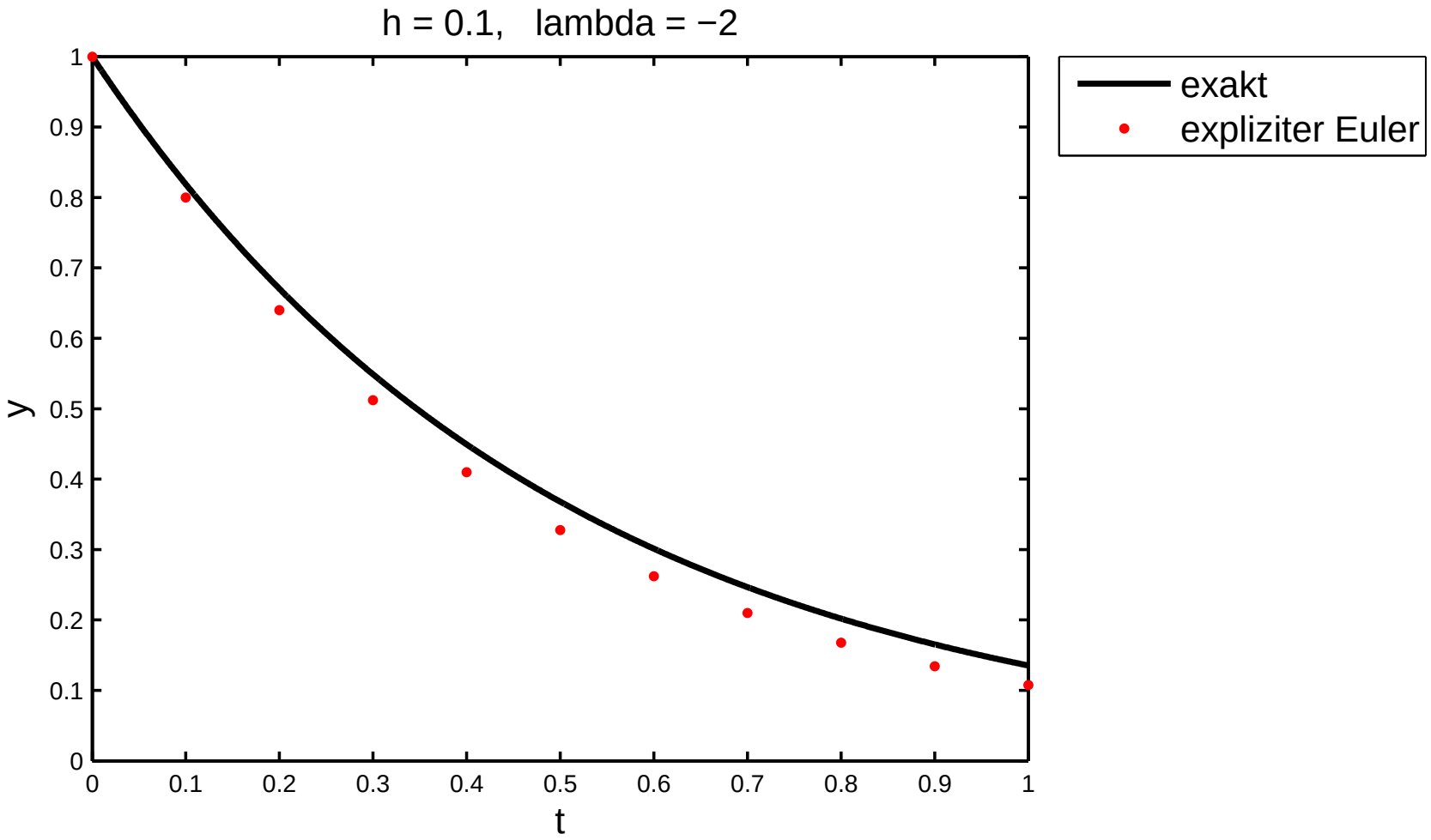
$$\begin{aligned} y'(t) &= \lambda y(t), \qquad t \in (0,1) \\ y(0) &= 1 \end{aligned}$$

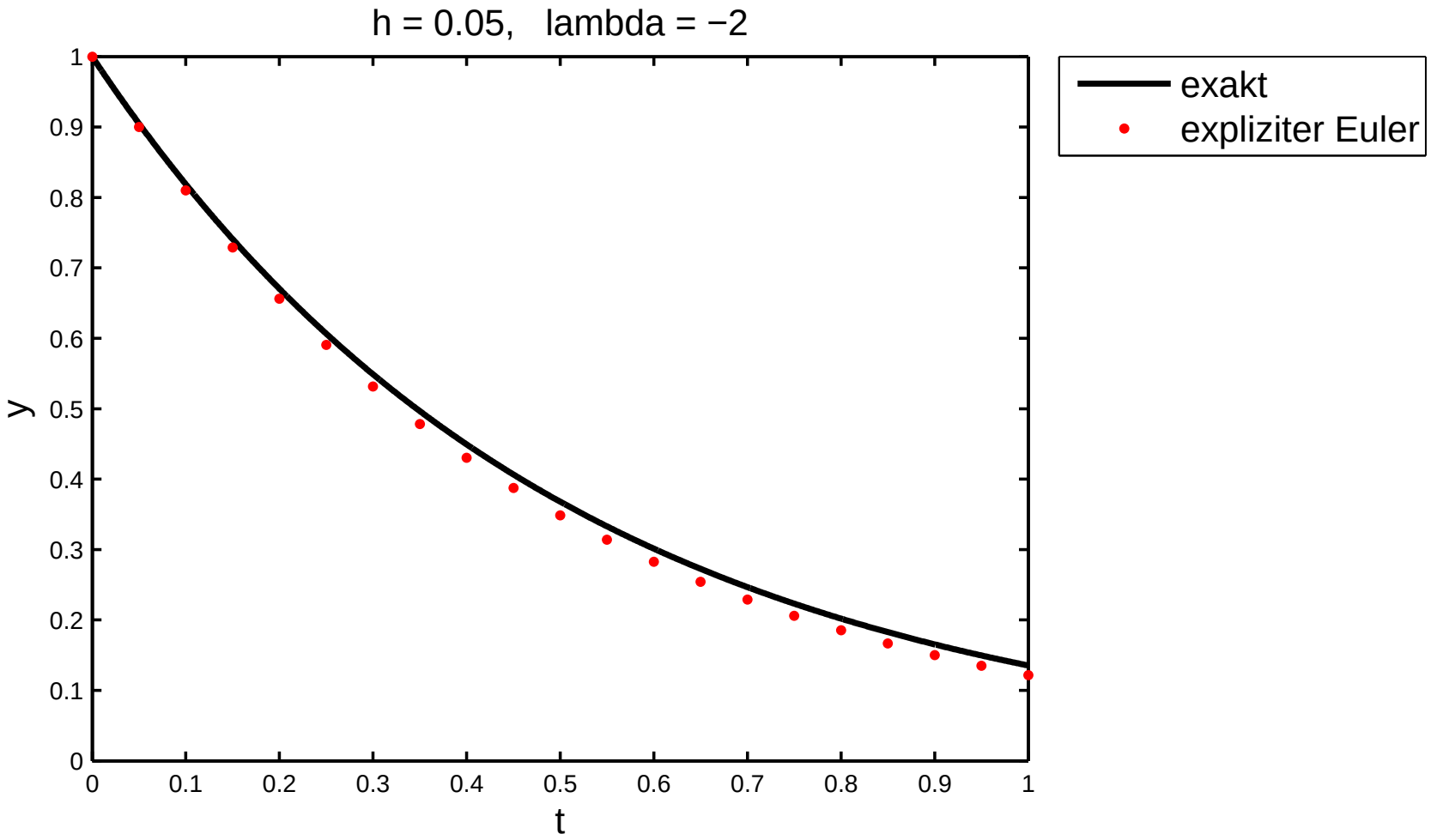
Exakte Lösung:

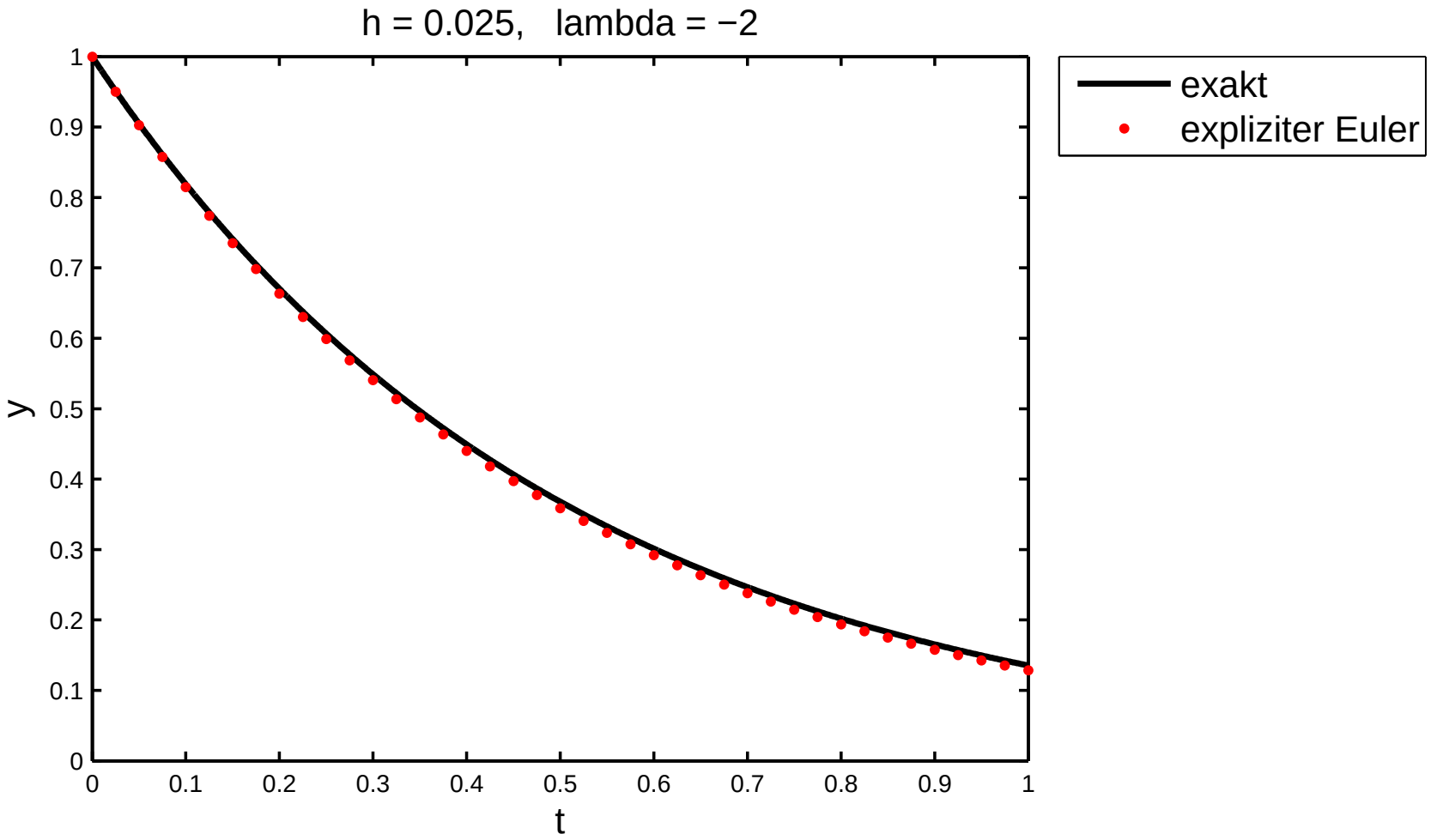
$$y(t) = e^{\lambda t}$$

Beispiel 1: explizites Euler-Verfahren für $\lambda = -2$

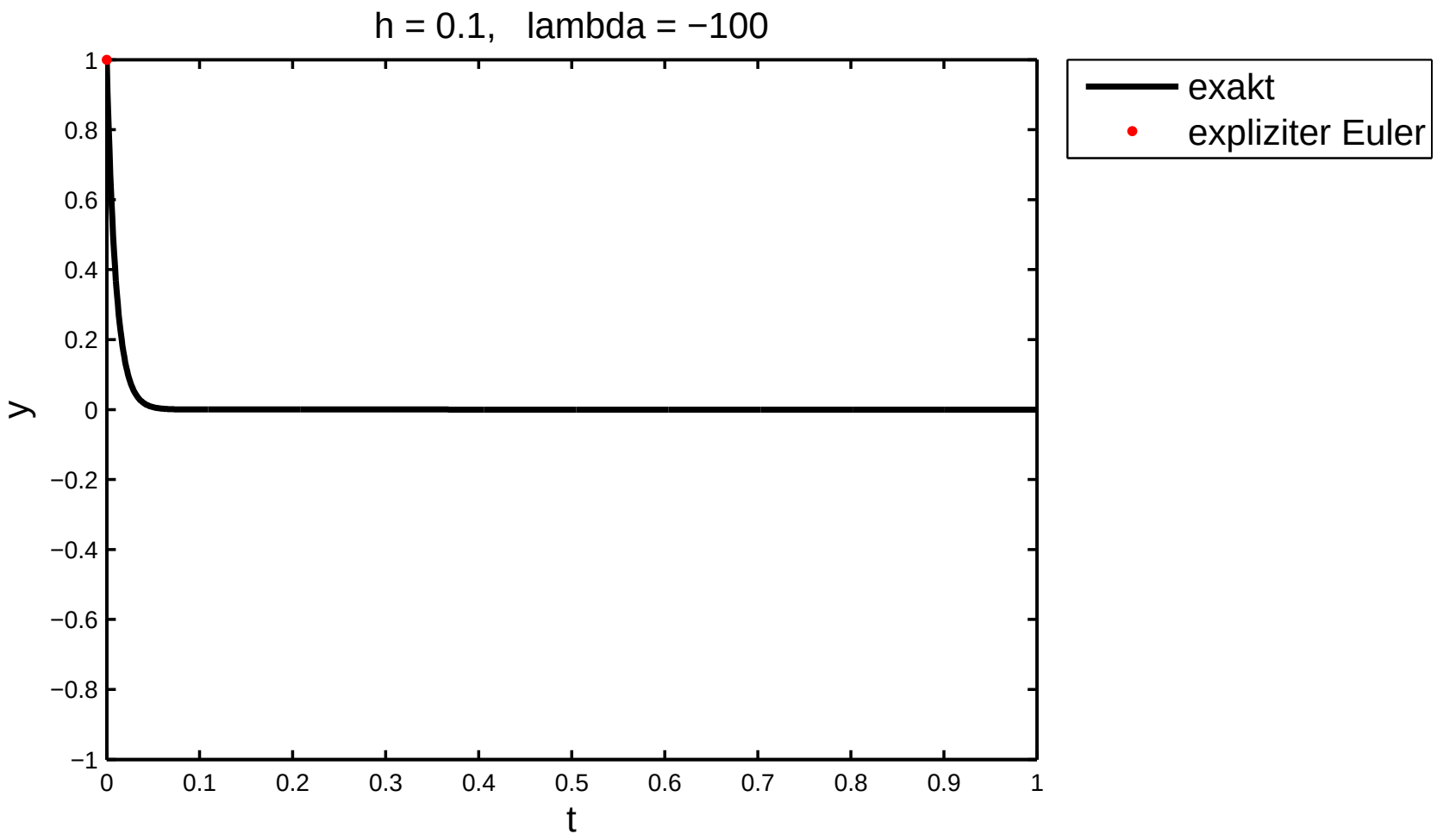

h = 0.2, lambda = −2
y
t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
exakt
expliziter Euler

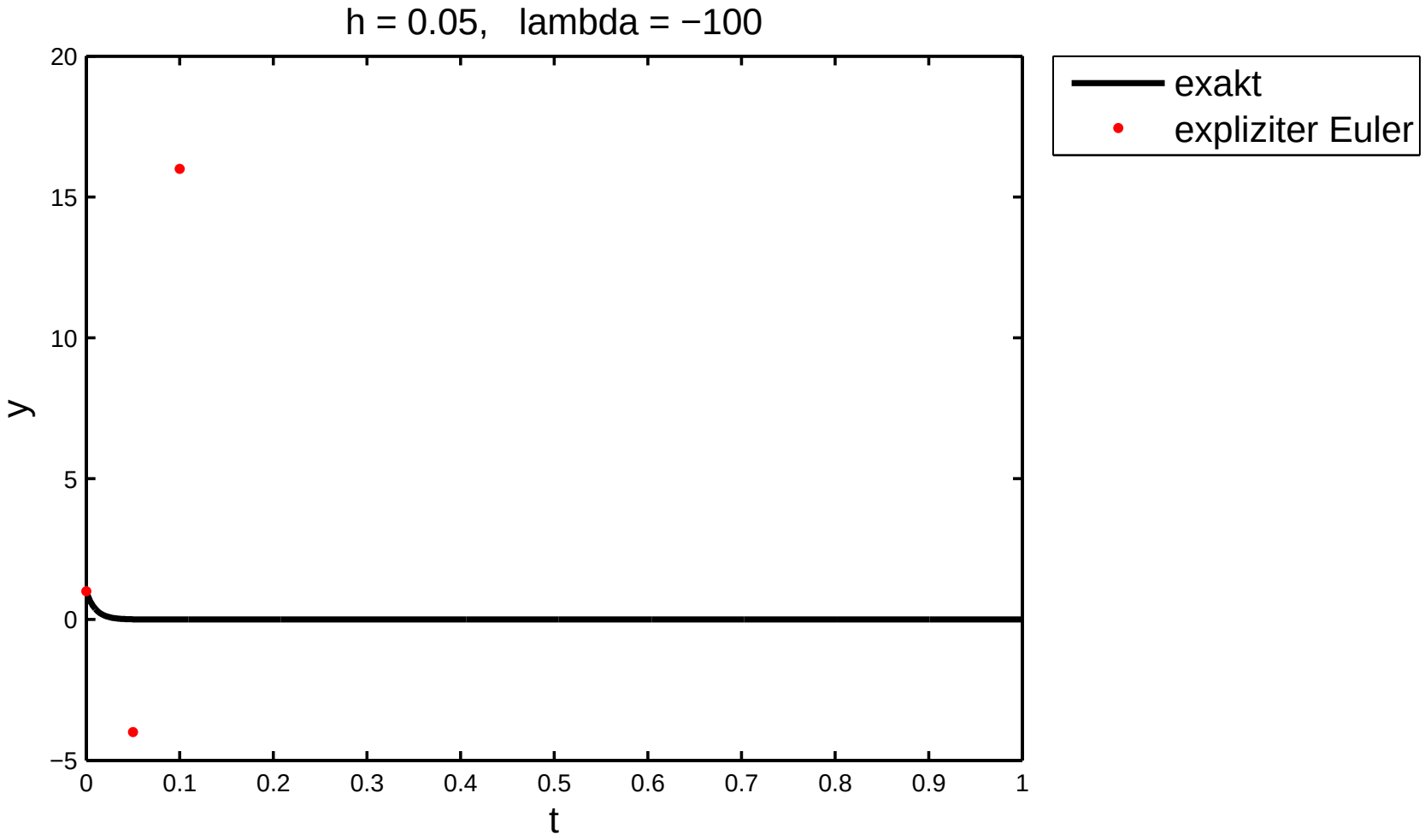

h = 0.1, lambda = −2
y
t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
exakt
expliziter Euler

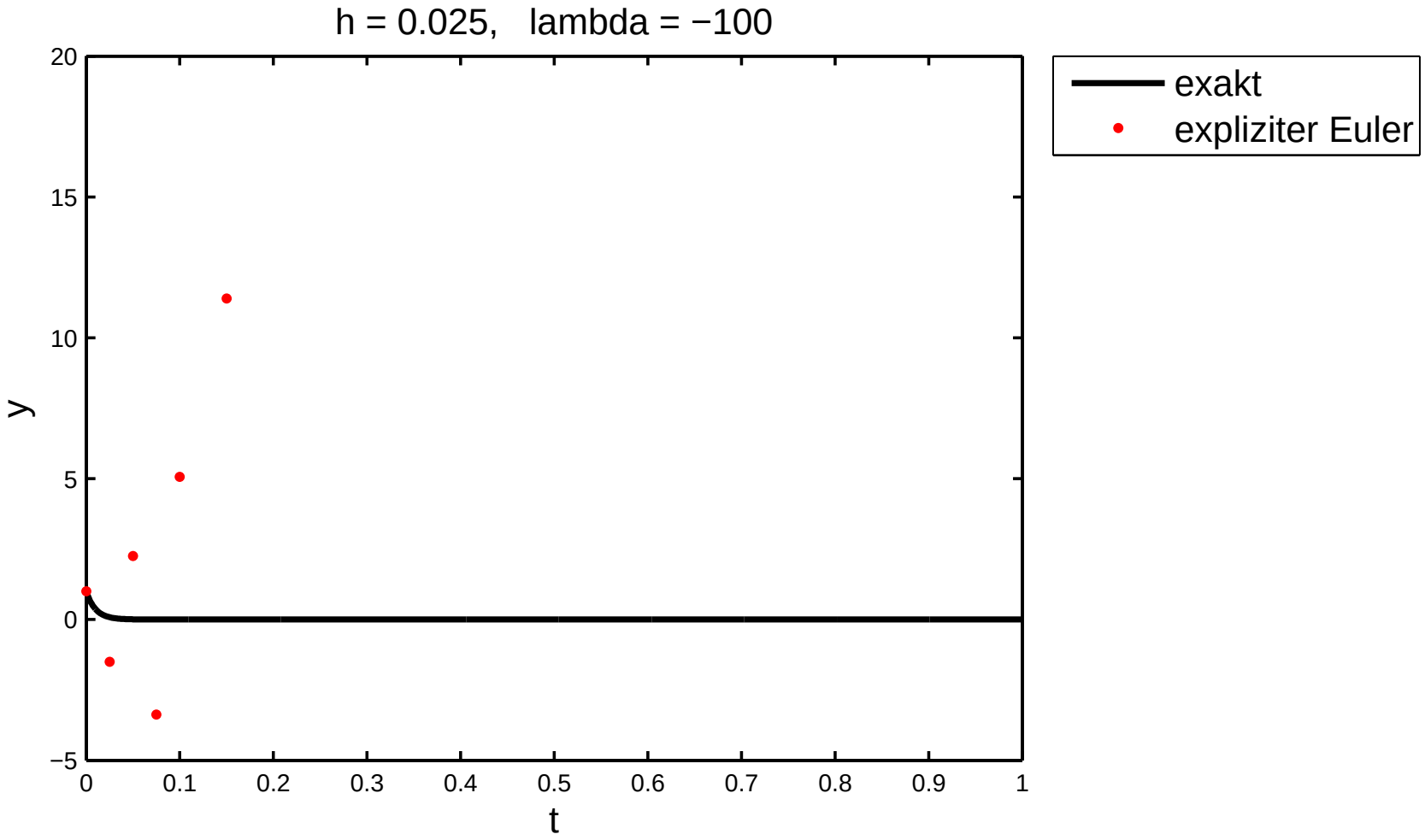

h = 0.05, lambda = −2
y
t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
exakt
expliziter Euler

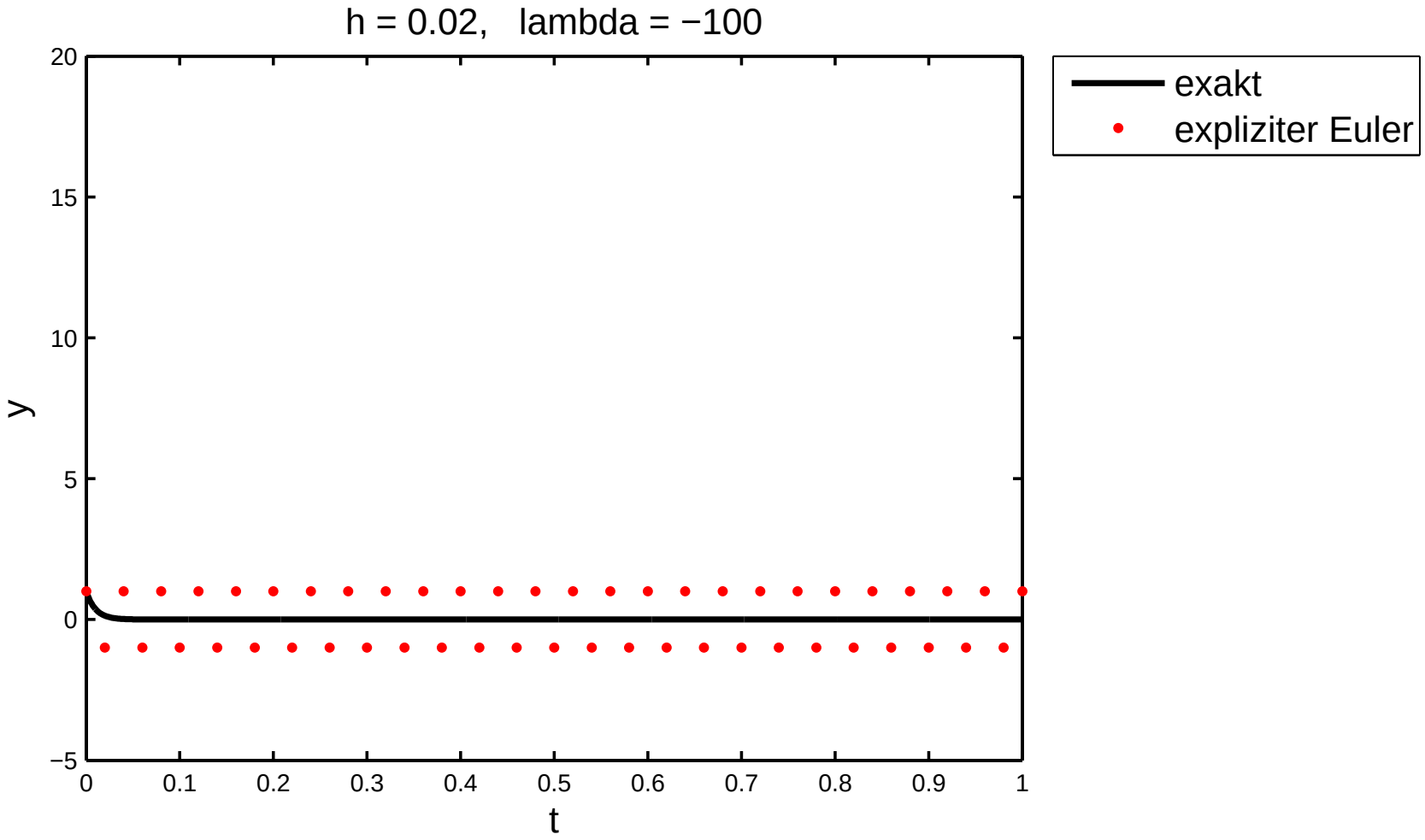

h = 0.025, lambda = −2
y
t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
exakt
expliziter Euler

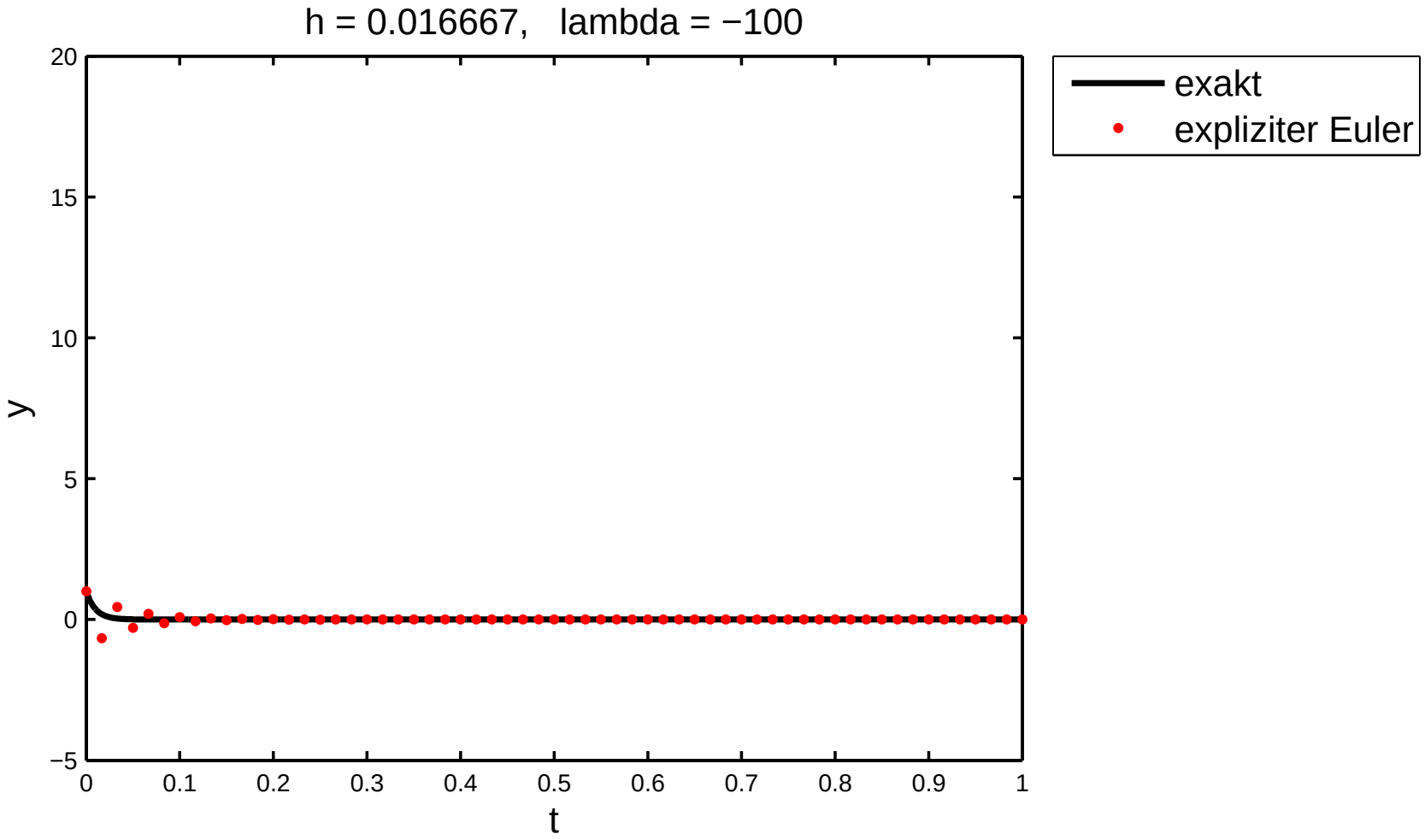
Beispiel 2: explizites Euler-Verfahren für $\lambda = -100$

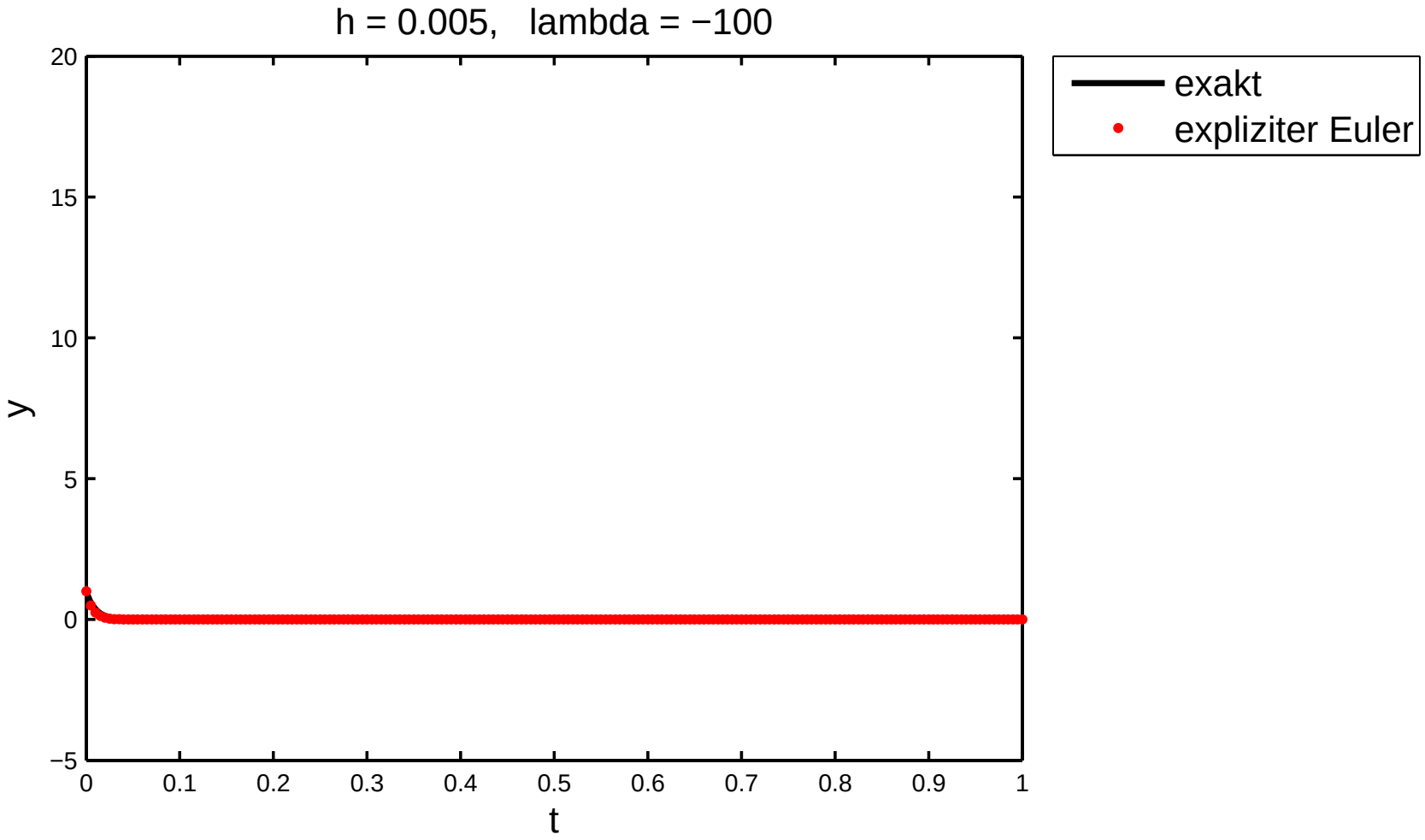

h = 0.1, lambda = −100
y
t
exakt
expliziter Euler
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


h = 0.05, lambda = −100
20
15
10
5
0
−5
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
exakt
expliziter Euler

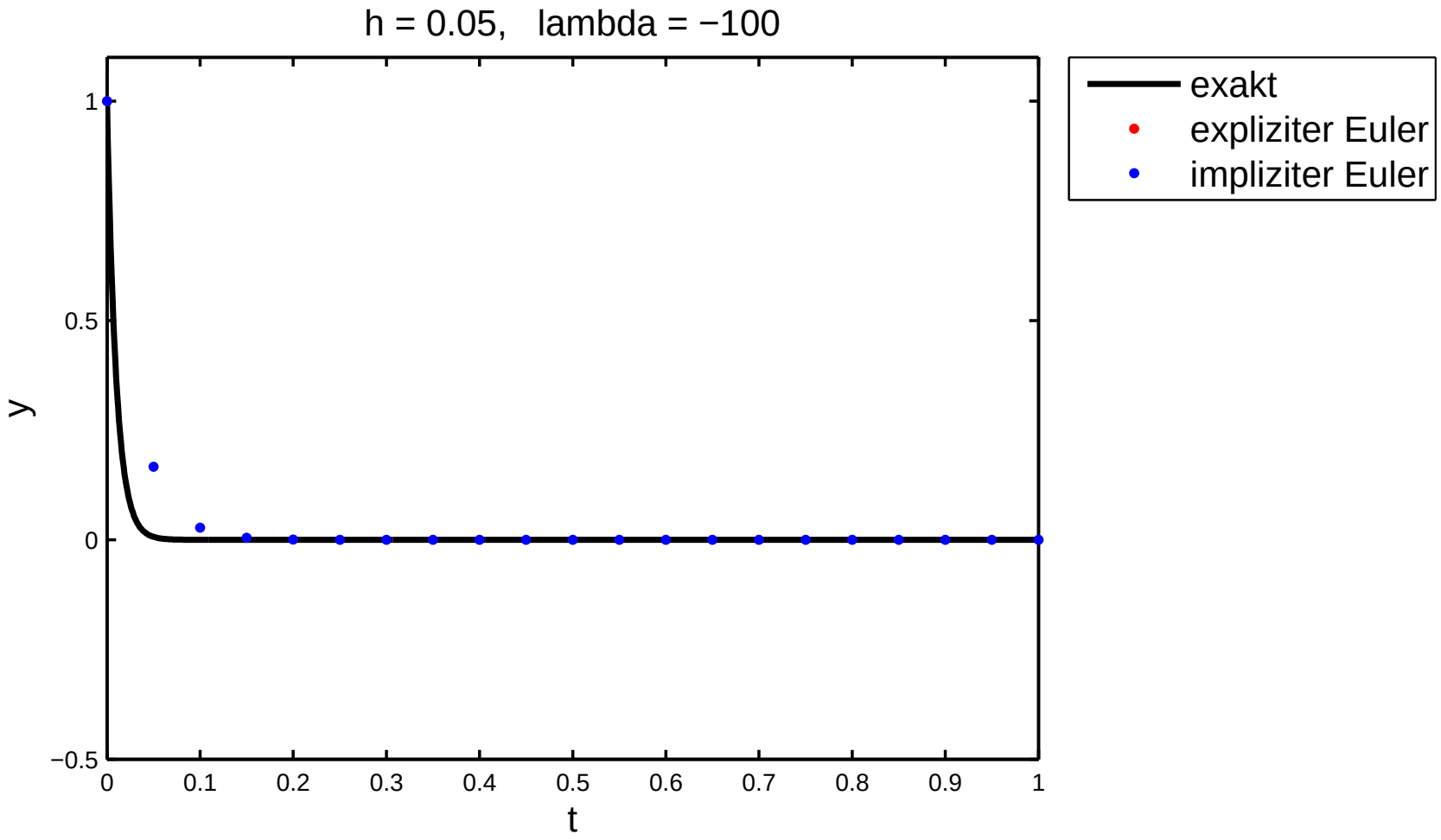

h = 0.025, lambda = −100
20
15
10
5
0
−5
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
exakt
expliziter Euler

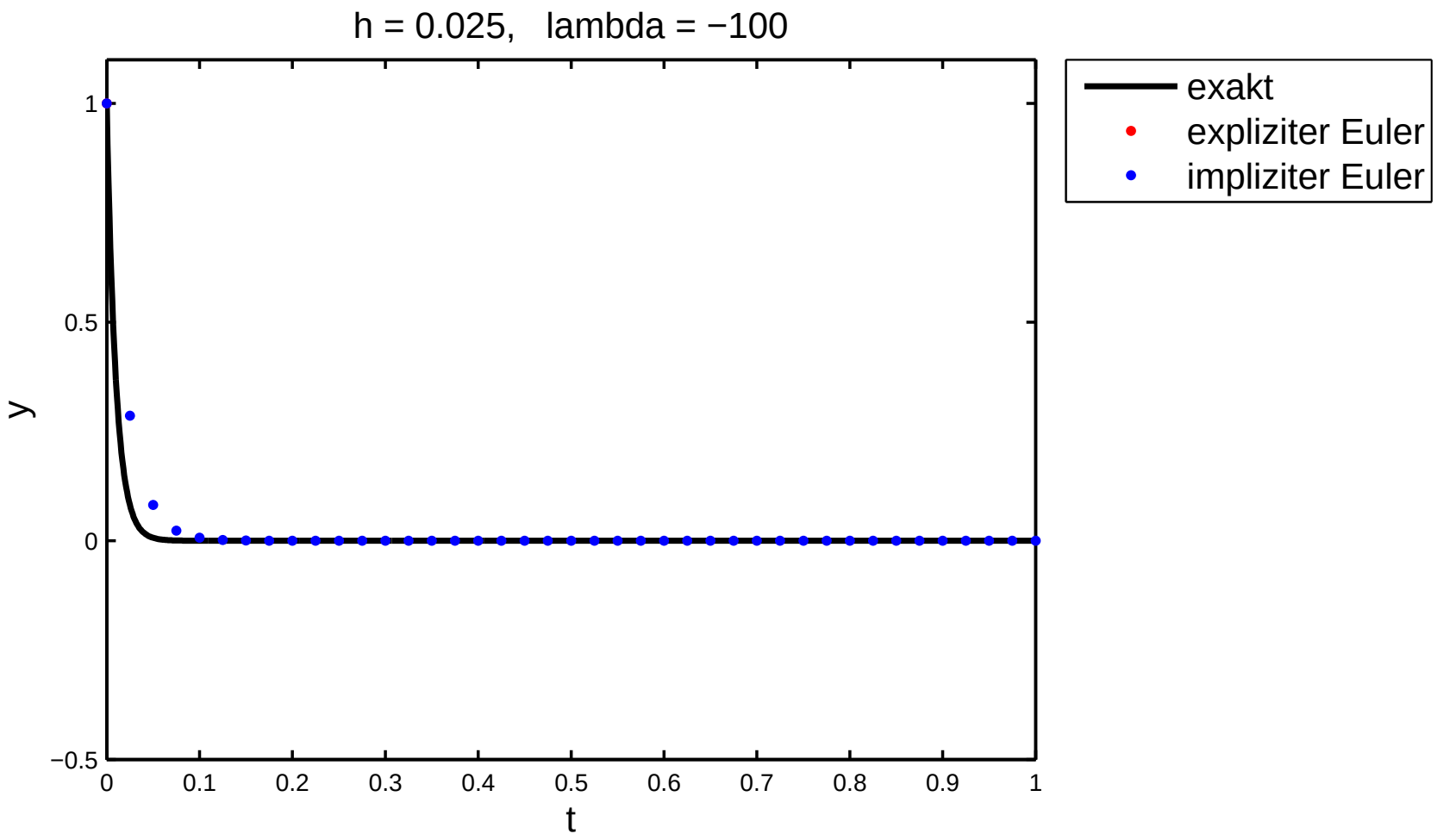

h = 0.02, lambda = −100
20
15
10
5
0
−5
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
exakt
expliziter Euler

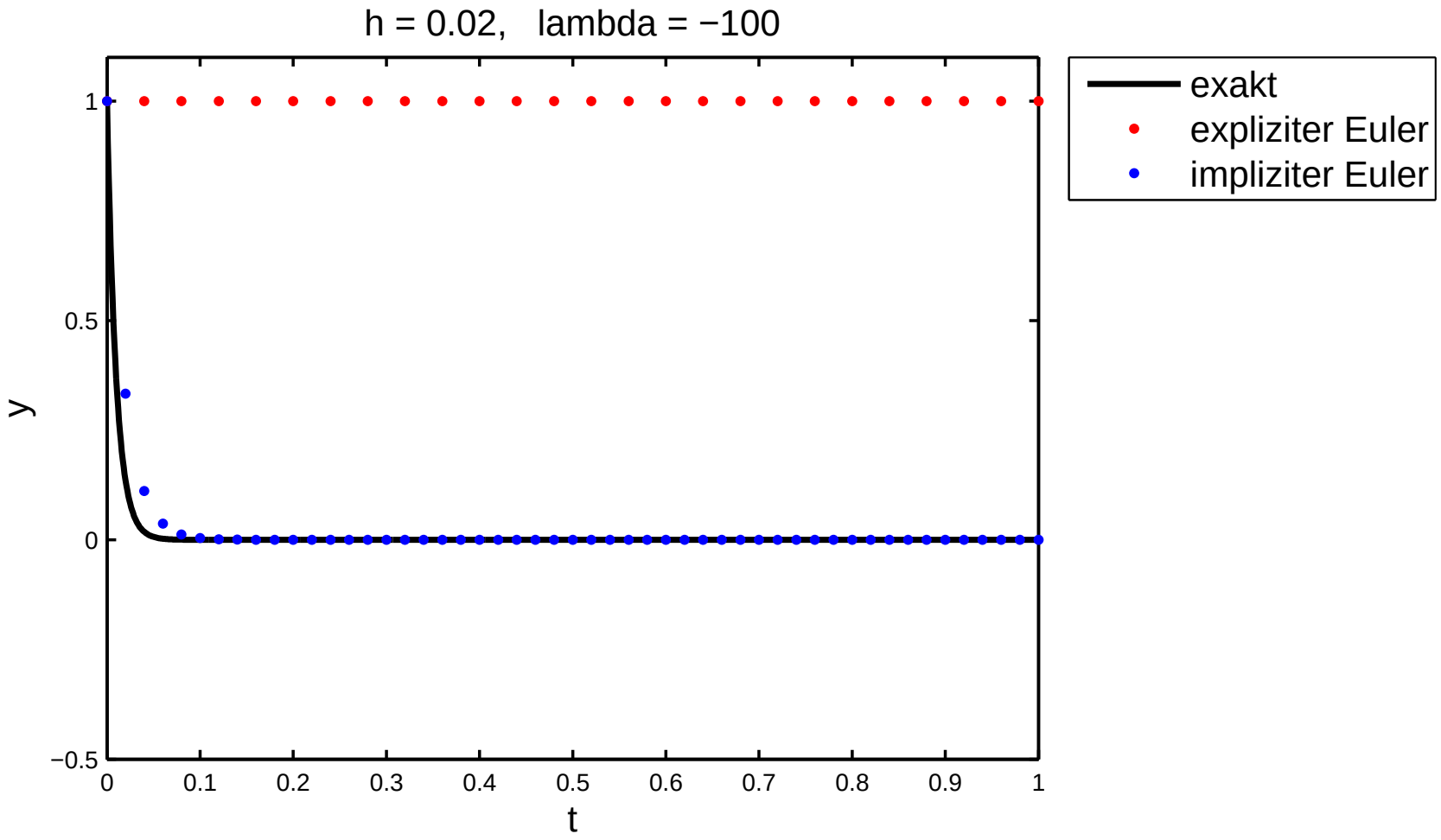

h = 0.016667, lambda = −100
20
15
10
5
0
−5
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
exakt
expliziter Euler

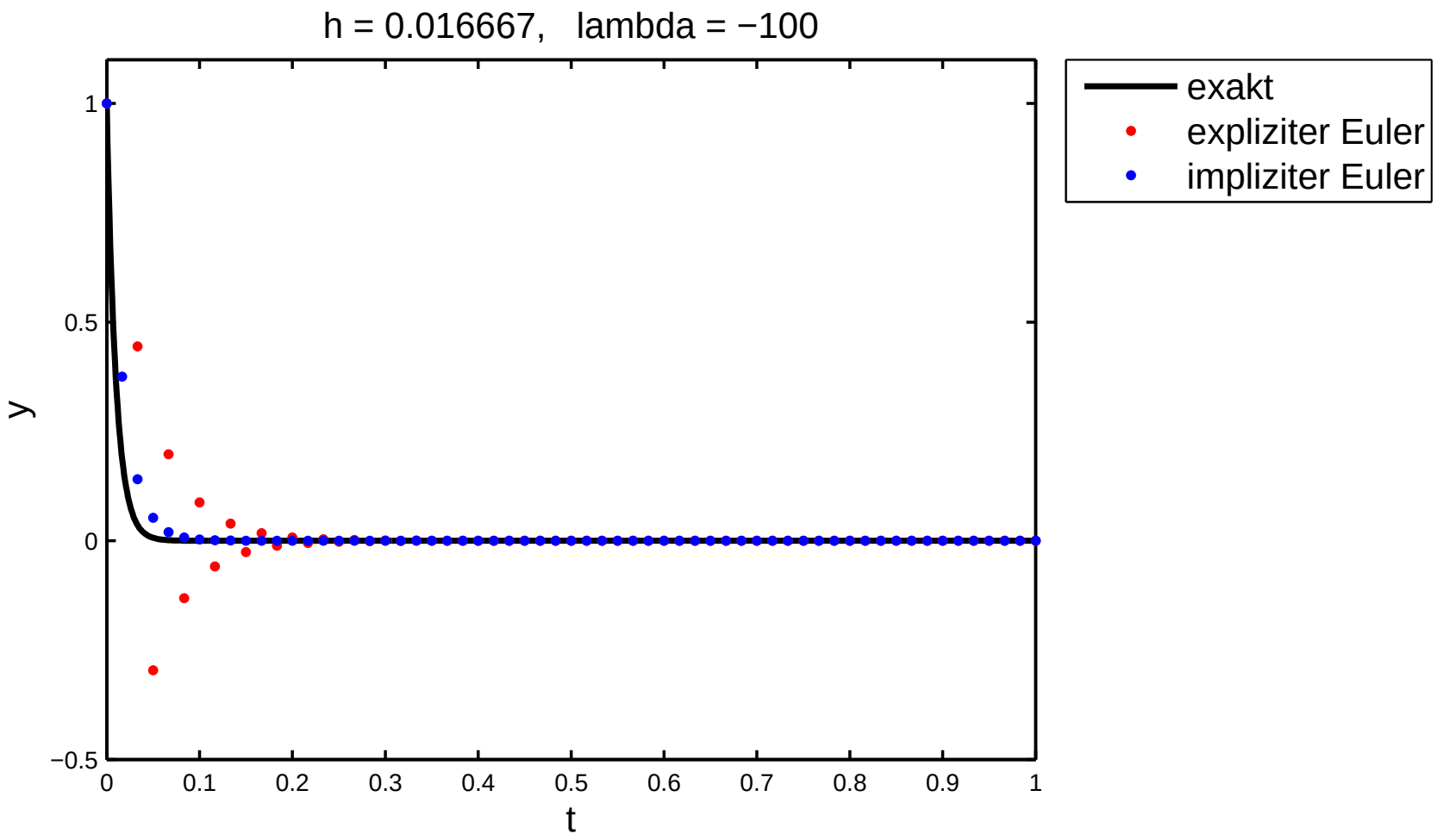

h = 0.005, lambda = −100
20
15
10
5
0
−5
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
exakt
expliziter Euler

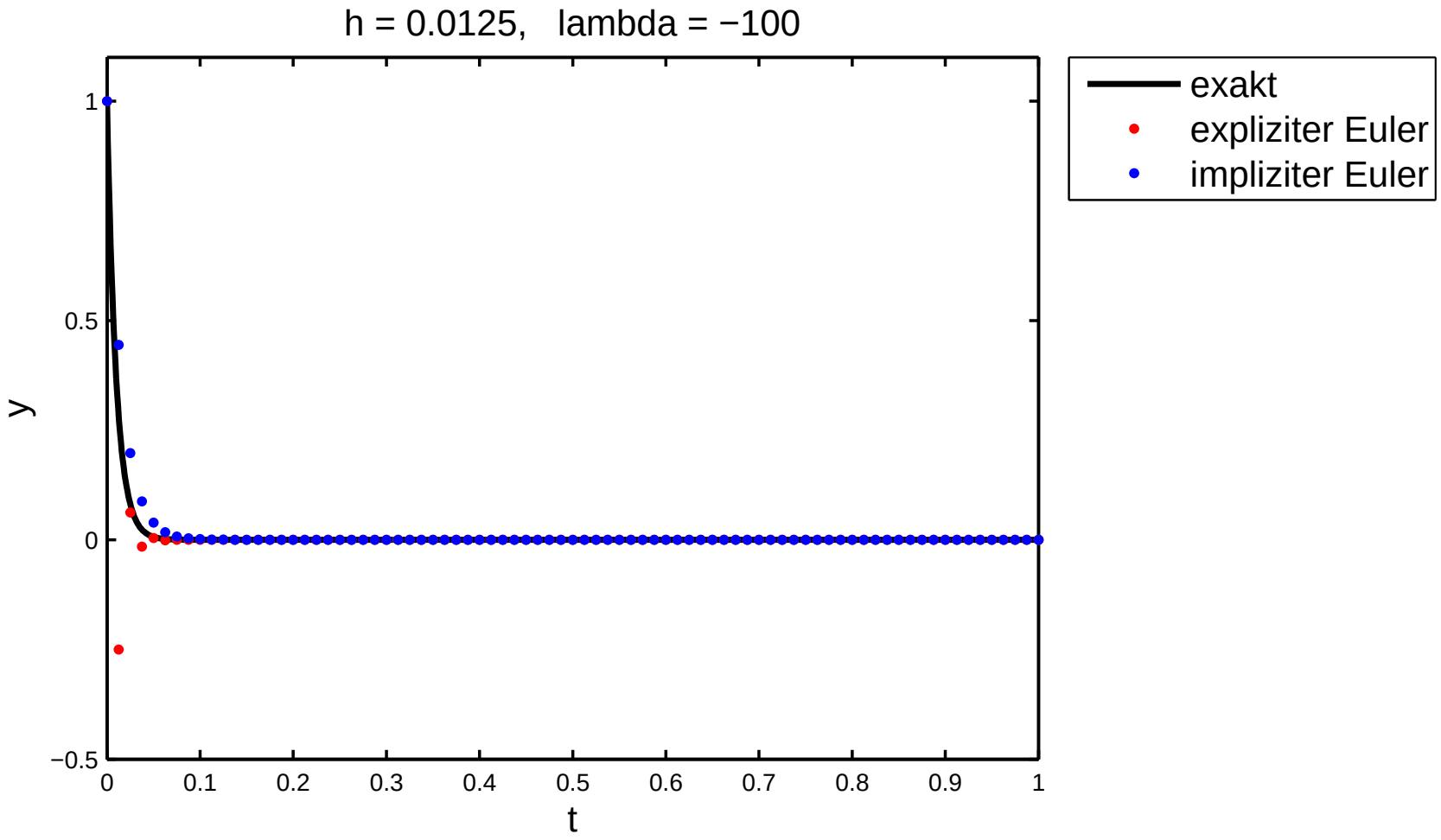
Beispiel 3: beide Euler-Verfahren für $\lambda = -100$

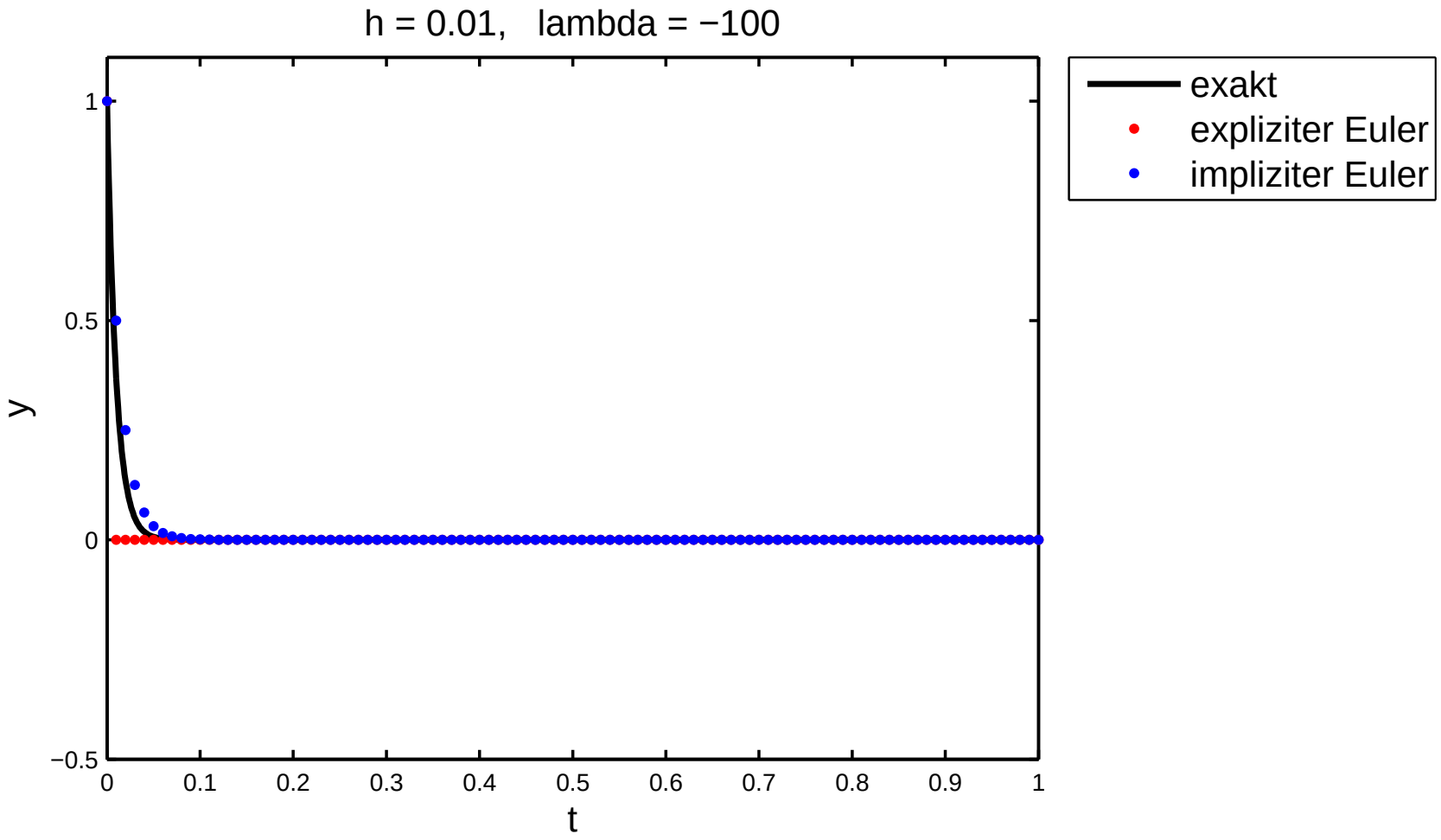

h = 0.05, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

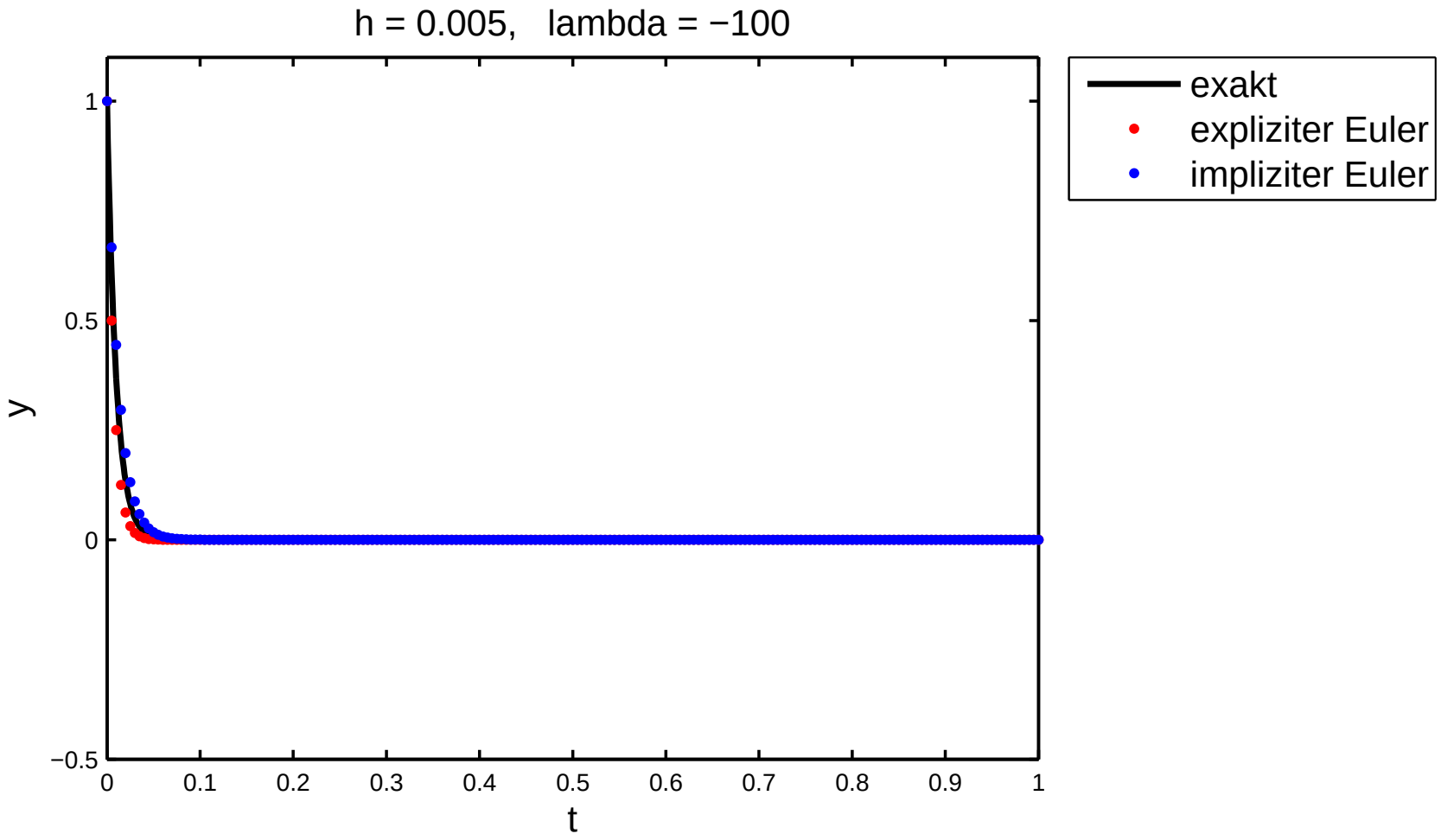

h = 0.025, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


h = 0.02, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

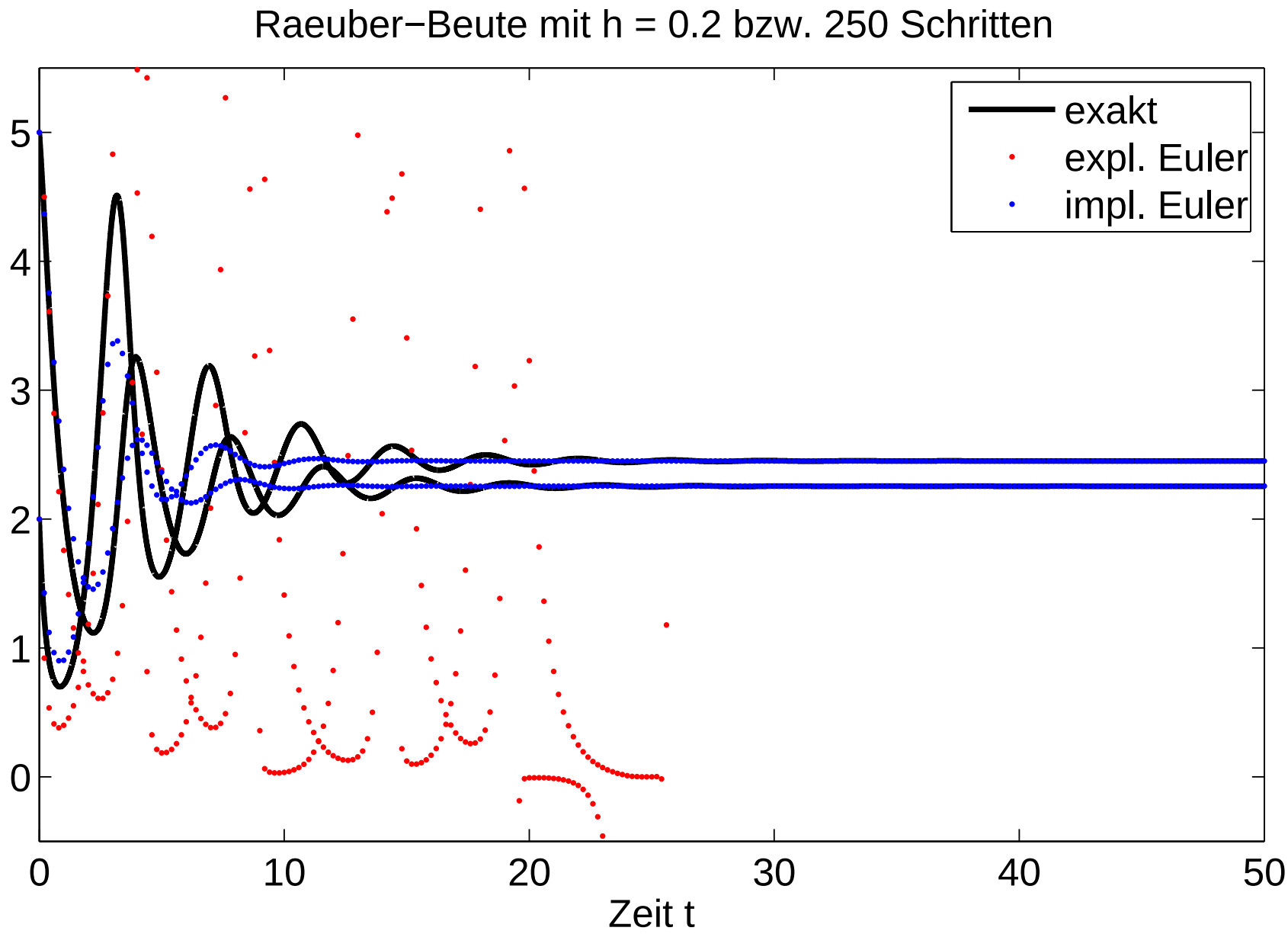

h = 0.016667, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

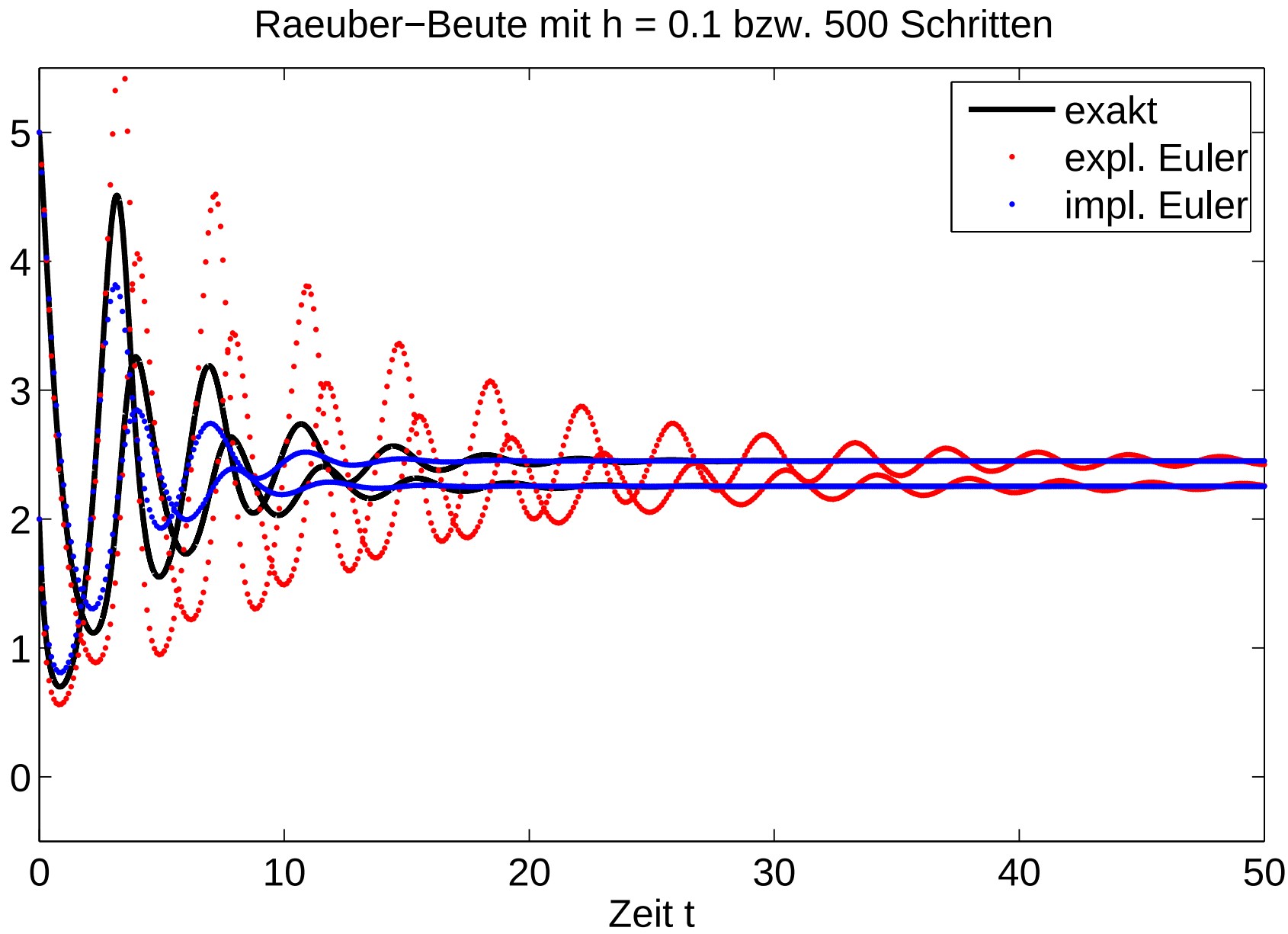

h = 0.0125, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

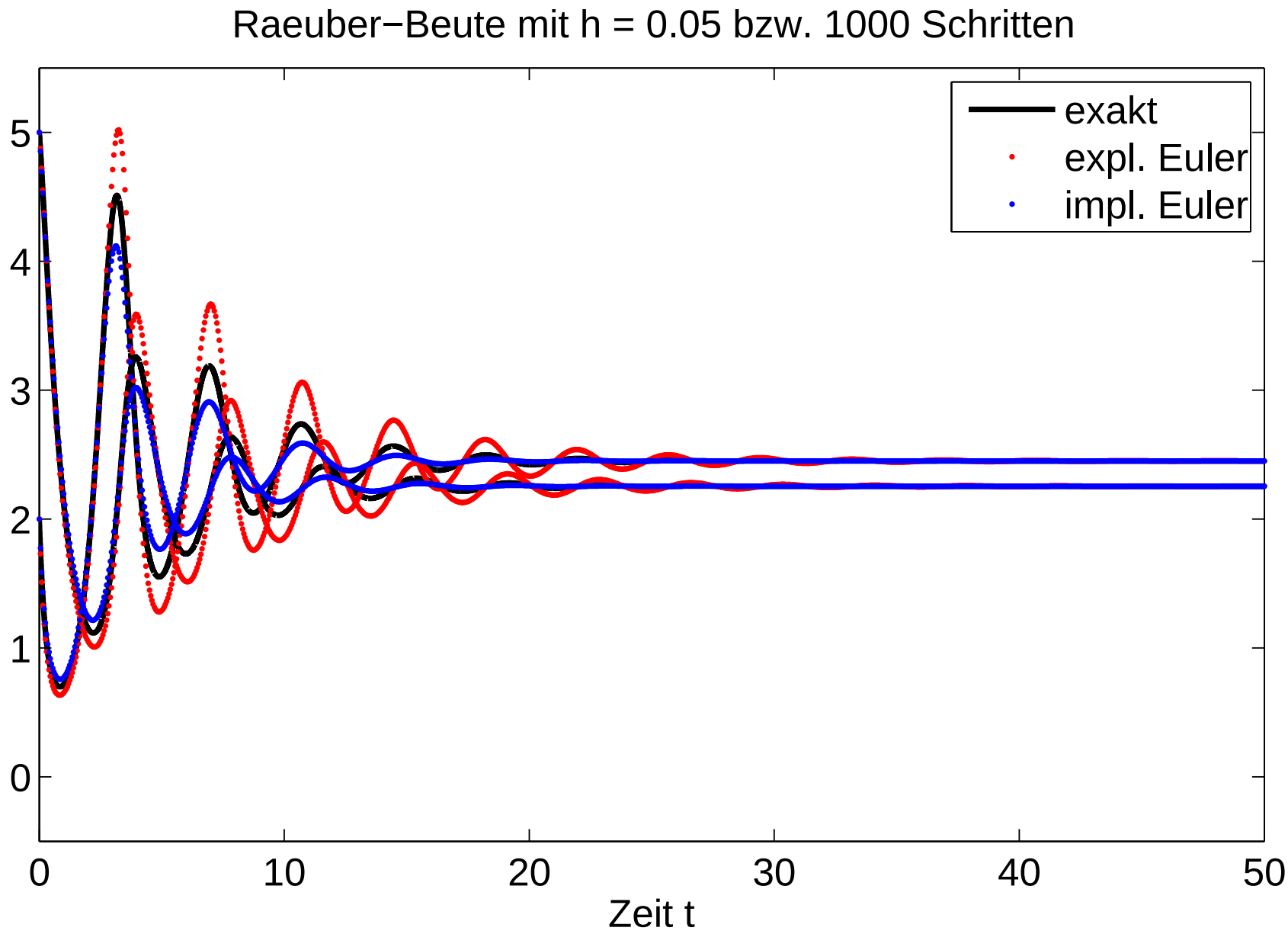

h = 0.01, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


h = 0.005, lambda = −100
exakt
expliziter Euler
impliziter Euler
y
t
1
0.5
0
−0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

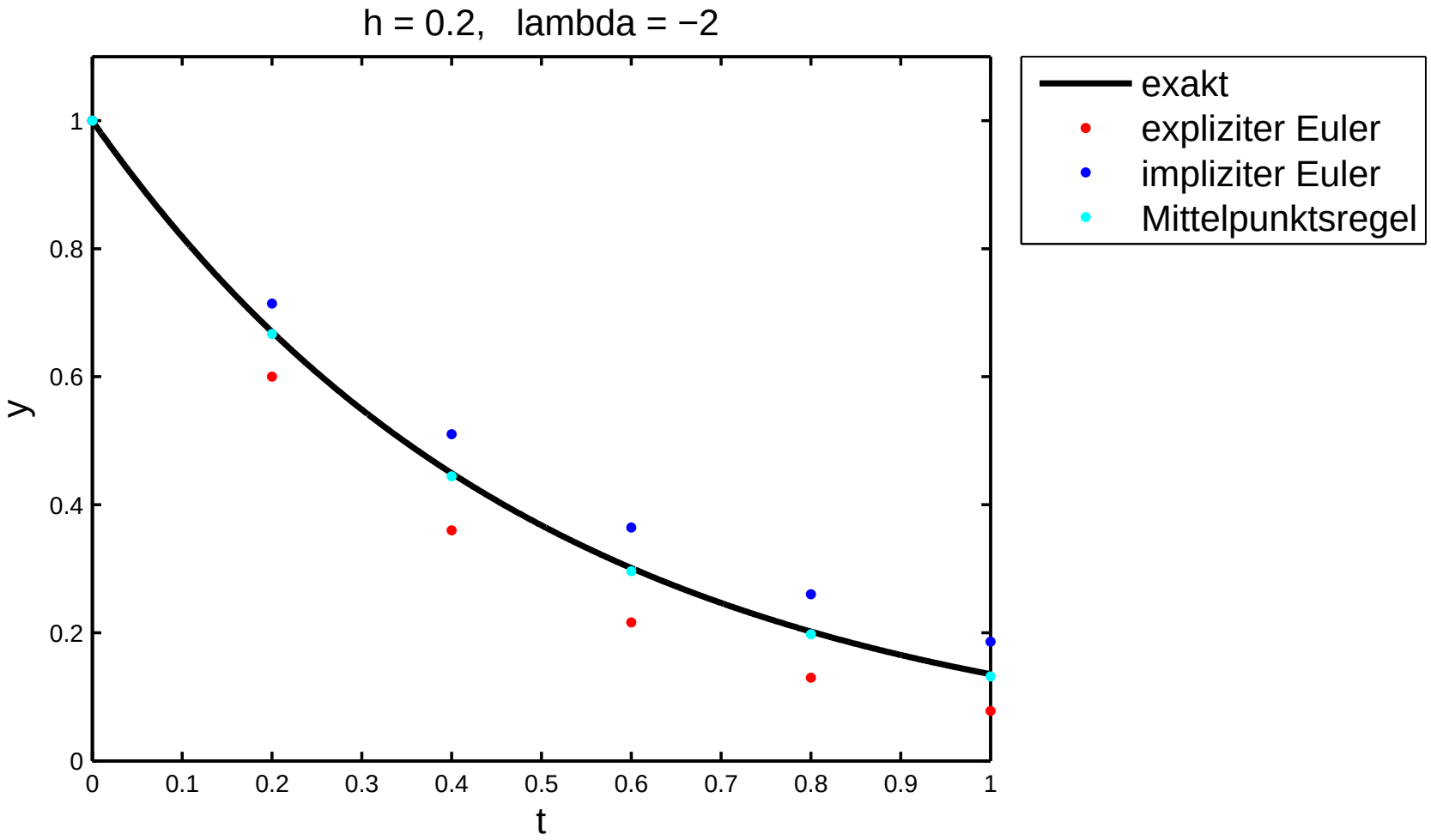
Beispiel 4: beide Euler-Verfahren angewandt auf ein Räuber-Beute-Modell

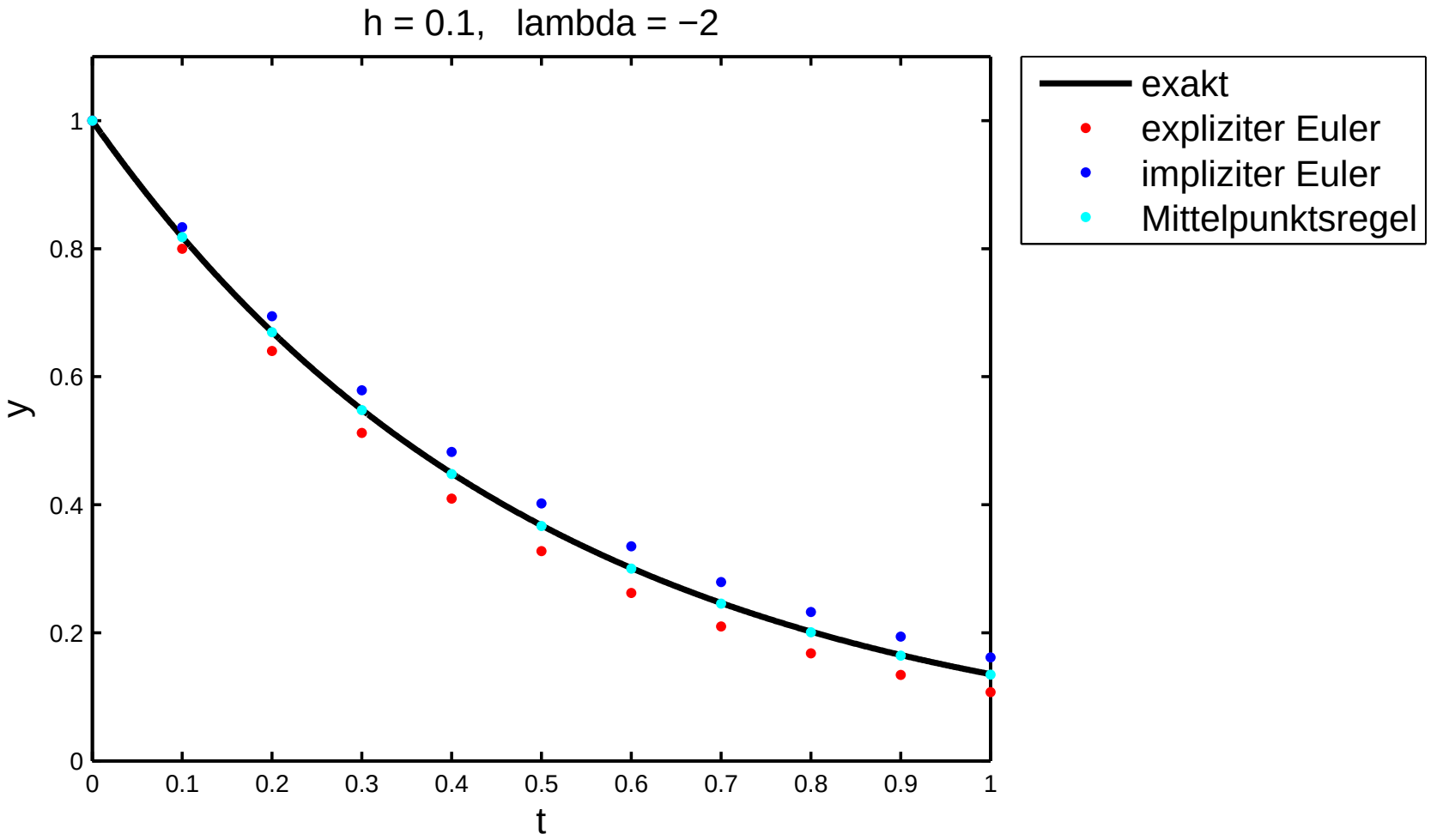

Raeuber−Beute mit h = 0.2 bzw. 250 Schritten
exakt
expl. Euler
impl. Euler
0
1
2
3
4
5
0
10
20
30
40
50
Zeit t

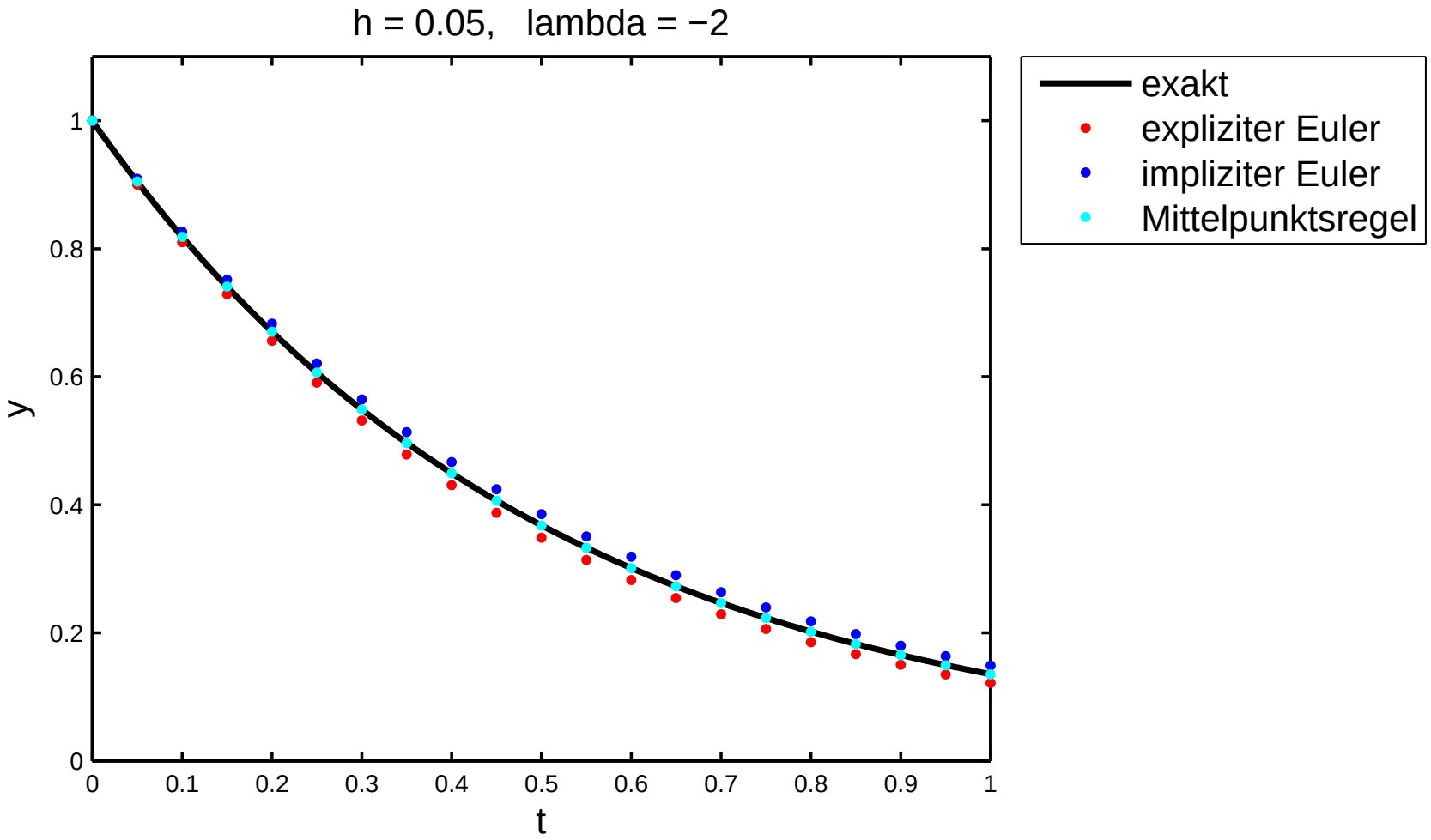

Raeuber−Beute mit h = 0.1 bzw. 500 Schritten
exakt
expl. Euler
impl. Euler
0
1
2
3
4
5
0
10
20
30
40
50
Zeit t

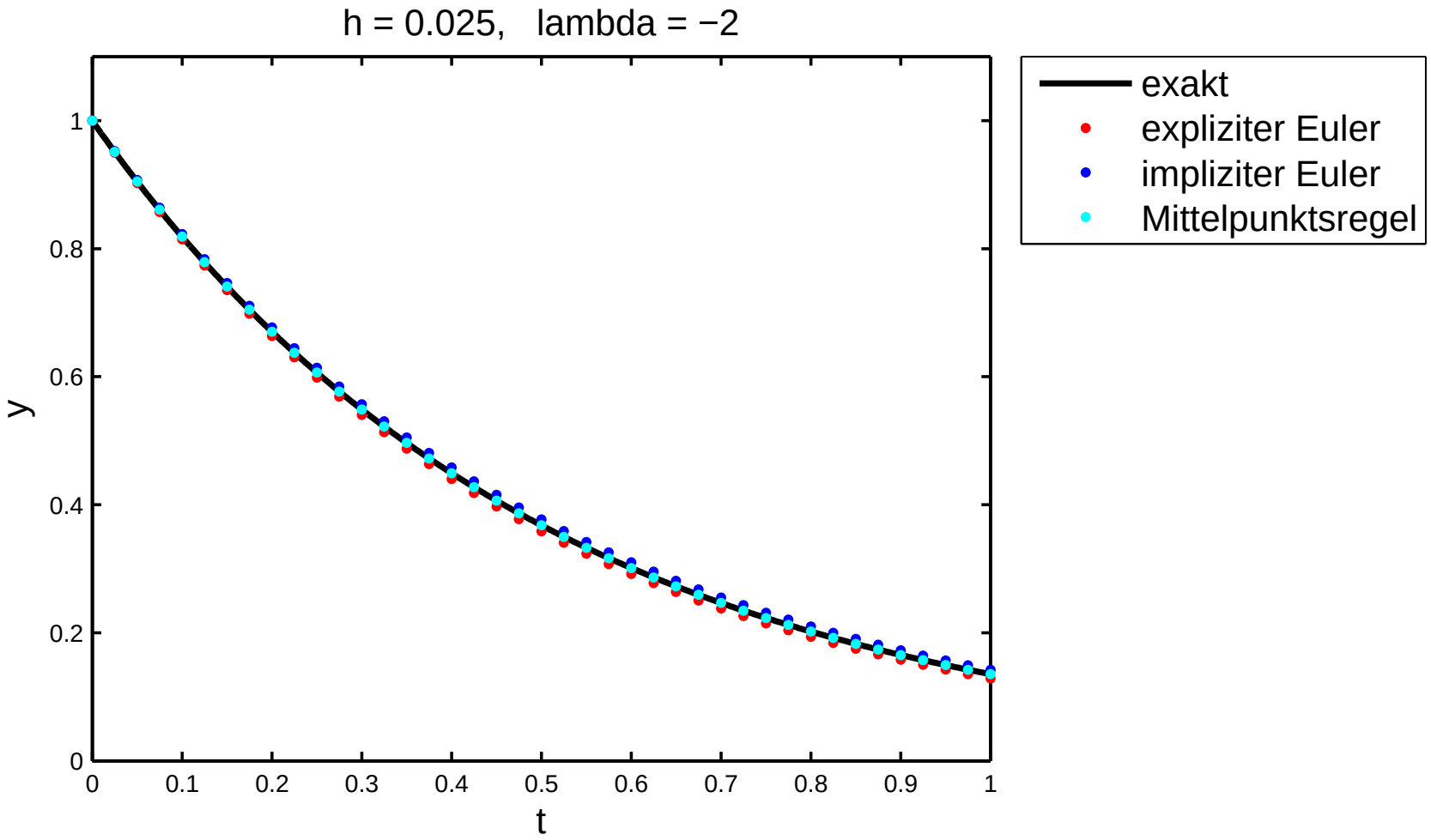

Raeuber−Beute mit h = 0.05 bzw. 1000 Schritten
exakt
expl. Euler
impl. Euler
0
1
2
3
4
5
0
10
20
30
40
50
Zeit t

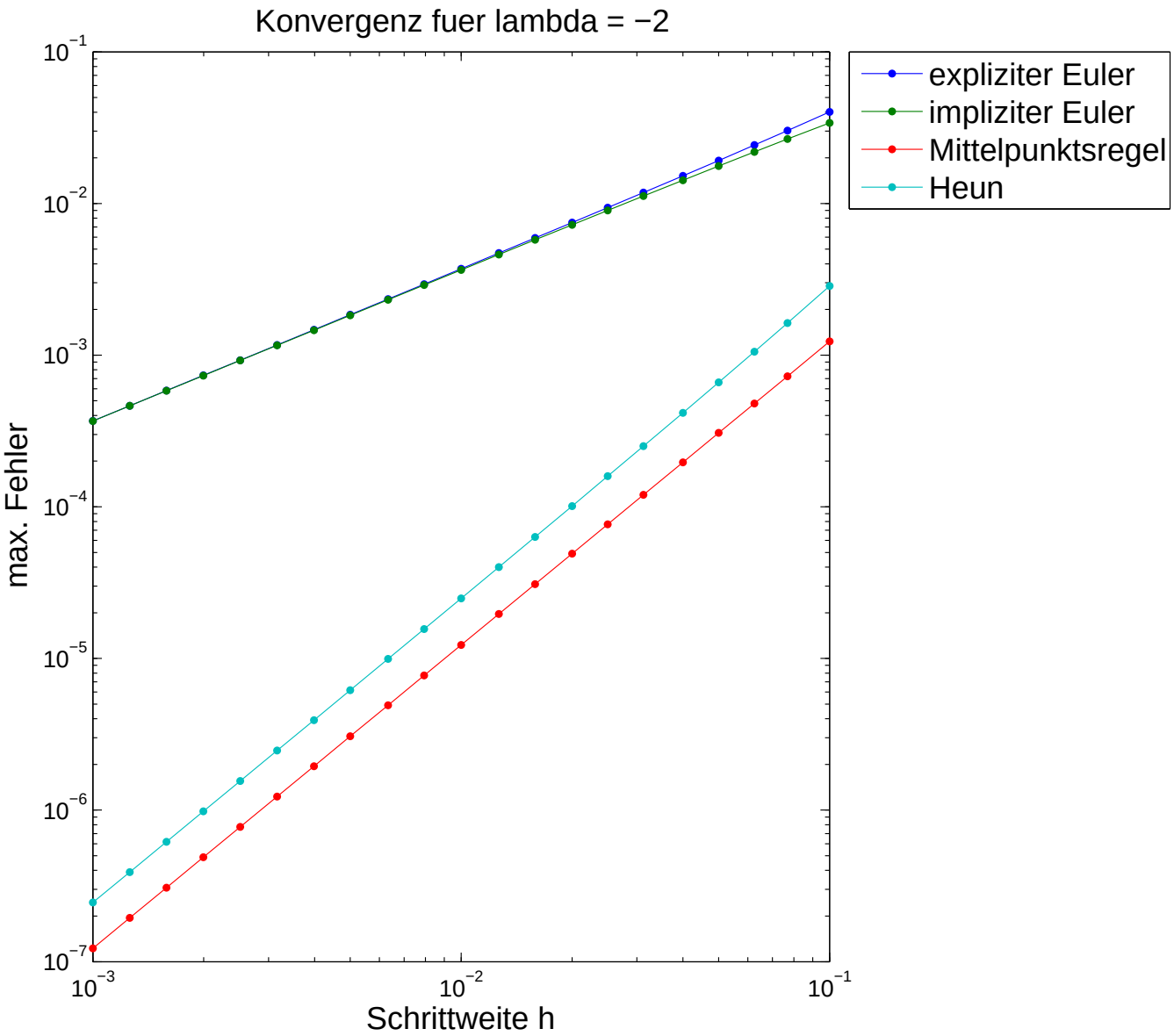
Beispiel 5: beide Euler-Verfahren und die Mittelpunktsregel für $\lambda = -2$


h = 0.2, lambda = −2
y
t
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9
exakt
expliziter Euler
impliziter Euler
Mittelpunktsregel


h = 0.1, lambda = −2
y
t
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9
exakt
expliziter Euler
impliziter Euler
Mittelpunktsregel


h = 0.05, lambda = −2
y
t
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9
exakt
expliziter Euler
impliziter Euler
Mittelpunktsregel


h = 0.025, lambda = −2
y
t
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9
exakt
expliziter Euler
impliziter Euler
Mittelpunktsregel


Konvergenz fuer lambda = −2
max. Fehler
Schrittweite h
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-3}
10^{-2}
10^{-1}
expliziter Euler
impliziter Euler
Mittelpunktsregel
Heun

Stabilität und Konvergenz der Euler-Verfahren

Tobias Jahnke

Karlsruher Institut für Technologie

Vorlesung *Quantitative Biologie*

Sommersemester 2012