A 15

(1) $x_m = x_{m-1} + \tau_m p_m$

(2) $r_m = r_{m-1} - \tau_m A p_m$

(3) $p_{m+1} = r_m + \mu'_{m+1} p_m$

Für $m=0$ ist nichts zu zeigen. Also Induktion $m-1 \rightsquigarrow m$

Orthogonalität:

$$r_{m-1}^H r_m \overset{(2)}{=} r_{m-1}^H (r_{m-1} - \tau_m A p_m)$$

$$= r_{m-1}^H r_{m-1} - \tau_m r_{m-1}^H A p_m$$

$$\overset{(3)}{=} S_m - \tau_m (p_m - \mu'_m p_{m-1})^H A p_m$$

$$= S_m - \tau_m p_m^H A p_m$$

$$= S_m - \frac{S_m}{S'_m} \cdot S'_m = 0$$

A-Konjugiert:

$$p_m^H A r_m = (A p_m)^H r_m \overset{(2)}{=} \frac{1}{\tau_m} (r_{m-1} - r_m)^H r_m$$

$$= -\frac{1}{\tau_m} r_m^H r_m = -\frac{S_{m+1}}{\tau_m} = -\mu'_{m+1} S'_m$$

Also:

$$p_m^H A p_{m+1} = p_m^H A (r_m + \mu'_{m+1} p_m)$$

$$= p_m^H A r_m + \mu'_{m+1} p_m^H A p_m$$

$$= -\mu'_{m+1} S'_m + \mu'_{m+1} S'_m = 0$$