A10

a) $4n$ b) $4n-2$

c) periodische RB $\Rightarrow$ $s'(a) = s'(b)$ $\Leftrightarrow$ $v_0 = v_n$

und $s''(a) = s''(b)$

(warum?)

$$\Leftrightarrow 2\left(\frac{1}{h_1}+\frac{1}{h_n}\right)v_0 + \frac{v_1}{h_1} + \frac{v_{n-1}}{h_n} = \underbrace{3\left(\frac{\delta y[x_0,x_1]}{h_1} + \frac{\delta y[x_{n-1},x_n]}{h_n}\right)}_{=:\, b_0}$$

LGS $\rightsquigarrow$

$$\begin{pmatrix} 2\left(\frac{1}{h_1}+\frac{1}{h_n}\right) & \frac{1}{h_1} & 0 & \cdots & 0 & \frac{1}{h_n} \\ \frac{1}{h_1} & 2\left(\frac{1}{h_1}+\frac{1}{h_2}\right) & \frac{1}{h_2} & \ddots & \vdots & \vdots \\ 0 & \frac{1}{h_2} & \ddots & \ddots & 0 & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \frac{1}{h_{n-1}} \\ 0 & & \ddots & 0 & \frac{1}{h_{n-2}} & 2\left(\frac{1}{h_{n-1}}+\frac{1}{h_n}\right) \\ \frac{1}{h_n} & 0 & \cdots & & 0 & \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \\ \vdots \\ \vdots \\ v_{n-1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ \vdots \\ b_{n-1} \end{pmatrix}$$

mit $b_i = 3\left(\frac{\delta y[x_{i-1},x_i]}{h_i} + \frac{\delta y[x_i,x_{i+1}]}{h_{i+1}}\right)$ $i > 0$

A11

a) zeige $v_1 = 3\,\delta y[x_0,x_1] = 3\,\frac{y_1}{h}$ (1)

$v_{n-1} = 3\,\delta y[x_{n-1},x_n] = -3\,\frac{y_{n-1}}{h}$ (2)

$n=1$: ✓ (warum?)

$n=2$: zeige $v_1 = 0$ $\overset{(1)}{\Rightarrow}$ $y_1 = 0$ $\Rightarrow$ $s \equiv 0$ (warum?)

$n=3$: zeige $v_1 = v_2 = 0$ [stelle LGS auf]

$\overset{(2)(3)}{\Rightarrow}$ $y_1 = y_2 = 0$ $\Rightarrow$ $s \equiv 0$

b) $n=4$ $\qquad s(x_2) = y_2 = c$

$\rightsquigarrow$ LGS für v_1, v_2, v_3 (Es gilt $s_3''(x_3) = s_4''(x_3)$)

$$\begin{pmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \frac{3c}{h} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$\rightsquigarrow$ LGS ist lösbar (warum?)

c) Einsetzen in b)

$\rightsquigarrow$ $v_1 = -v_3 = \frac{3}{4} \wedge v_2 = 0$

$\overset{(1)+(2)}{\Longrightarrow}$ $s(x_1) = s(x_2) = \frac{1}{4}$