Numerical methods in mathematical finance

Lecture, winter semester 2023/24, 4+2 SWS, 8 ECTS
https://www.math.kit.edu/ianm3/lehre/numamathfin2023w/en

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An option is a contract which gives its owner the right to buy or sell an underlying asset at a future time at a fixed price. The underlying asset is often a stock of a company, and since its value varies randomly, computing the fair price of the corresponding option is an important and interesting problem which yields a number of mathematical challenges. This lecture provides an introduction to a number of models for option pricing. The main goal, however, is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way. The following topics will be treated:

- Mathematical models for pricing stock options
- Numerical methods for stochastic differential equations
- Monte Carlo simulation and multilevel Monte Carlo methods
- Monte Carlo integration and quasi-Monte Carlo methods
- Finite difference methods for parabolic partial differential equations
- Numerical methods for free boundary value problems

Prerequisites: Participants have to be familiar with

- probability theory (e.g. lecture “Wahrscheinlichkeitstheorie” at KIT),
- the Itô integral, the Itô formula, stochastic differential equations,
- basics of numerical mathematics (e.g. lectures “Numerische Mathematik 1+2” at KIT), and
- programming in MATLAB (preferred) or PYTHON.

A brief introduction to stochastic differential equations is provided here:

https://www.math.kit.edu/ianm3/lehre/numamathfin2023w/media/intro-to-sdes.pdf

Students who do not know the Itô integral and the Itô formula yet can use these notes to learn the basics before the semester starts.

Knowledge about stocks, options, arbitrage and other aspects from mathematical finance is not required, because the lecture will provide a short introduction to these topics.

Format: The course consists of a lecture and a problem class, both given in English. In the problem class the students will solve small exercises which illustrate the contents of the lecture. Moreover, participants are supposed to write short MATLAB or PYTHON programs in order to test and apply the algorithms which will be presented in the lecture.

Exams: Oral. The date will be chosen together with the students.

References

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[2] Michael B. Giles. Multilevel Monte Carlo path simulation. *Operations Research*, 56(3):607–617, 2008.

[3] Michael Günther and Ansgar Jüngel. *Finanzderivate mit MATLAB. Mathematische Modellierung und numerische Simulation.* Vieweg, Wiesbaden, 2nd ed. edition, 2010.

[4] Desmond J. Higham. *An introduction to financial option valuation.* Cambridge University Press, Cambridge, 2004. Mathematics, stochastics and computation.

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[7] Steven E. Shreve. *Stochastic calculus for finance. II: Continuous-time models.* Springer Finance. Springer, 2004.

[8] J. Michael Steele. *Stochastic calculus and financial applications.* Number 45 in Applications of Mathematics. Springer, New York, NY, 2001.