(12) Cholesky-Zerlegung

Geg: $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & \gamma \\ 3 & \beta & \alpha \end{pmatrix}$, $\alpha, \beta, \gamma \in \mathbb{R}$

Cholesky-Zerl: $A = LL^T$, $L \in \mathbb{R}^{n \times n}$ untere Δ-Mat

Berechnung:

$$a_{ij} = \sum_{k=1}^{j} l_{ik} l_{jk}, \quad i \geq j$$

$$\rightsquigarrow \quad l_{ij} = \begin{cases} 0, & i < j \\ \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}, & i = j \\ \frac{1}{l_{jj}}\left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk}\right), & i > j \end{cases}$$

Also: (1) $l_{11} = \sqrt{a_{11}} = 1$

(2) $l_{21} = \frac{1}{1}(1 - 0) = 1$

$l_{22} = \sqrt{2 - 1} = 1$

(3) $l_{31} = \frac{1}{1}(3 - 0) = 3$

$l_{32} = \frac{1}{1}(\beta - 3) = \beta - 3$

$l_{33} = \sqrt{\alpha - 9 - (\beta - 3)^2} = X$

$$\rightsquigarrow \begin{pmatrix} 1 & & 0 \\ 1 & 1 & \\ 3 & \beta-3 & X \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ & 1 & \beta-3 \\ 0 & & X \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & \gamma \\ 3 & \beta & \alpha \end{pmatrix}$$

Zunächst: LL^T symm $\Rightarrow$ A symm, falls Chol.-Zerl. ex.

$\Rightarrow \gamma = \beta$

Es gilt: A sym. pos. def

$\Leftrightarrow A = LL^T$, L untere Δ-Matrix mit $L_{ii} > 0$, $i = 1, \dots, n$

$X \in \mathbb{R}_{>0} \Leftrightarrow \alpha - 9 - (\beta - 3)^2 > 0$

$\Leftrightarrow \alpha > 9 + (\beta - 3)^2$