

⑤ LR-Zerlegung (Stabilität)

a) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 10^{-3} & 1 \\ 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 1 & 10^{-3} & 2 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \cdot 10^3$

Speicherplatz

$\sim \begin{pmatrix} 1 & 0 & -1 \\ 1 & 10^{-3} & 2 \\ 1 & 10^{-3} & -1999 \end{pmatrix}$ Also $A = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1000 & 1 \end{pmatrix}}_{=L} \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0,001 & 2 \\ 0 & 0 & -1999 \end{pmatrix}}_{=R}$

Löse: $Ax = b = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \Leftrightarrow L \overset{=y}{R} x = b \Leftrightarrow \begin{cases} Ly = b \\ Rx = y \end{cases}$

Vorwärtssubst: $\underline{y}_1 = 0, \underline{y}_2 = 1 - 0 = \underline{1}, \underline{y}_3 = 1 - 1000 - 0 = \underline{-999}$

Rückwärtssubst:

$\underline{x}_3 = \frac{+999}{+1999}, \underline{x}_2 = \frac{1}{10^{-3}} \left(1 - \frac{2 \cdot 999}{1999} \right) = 10^3 \cdot \left(\frac{1999 - 1998}{1999} \right) = \frac{1000}{1999}$

$\underline{x}_1 = \frac{1}{1} \cdot \left(0 + \frac{999}{1999} \right) = \frac{999}{1999}$

$\rightarrow \underline{x} = \frac{1}{1999} \begin{pmatrix} 999 \\ 1000 \\ 999 \end{pmatrix} \approx \begin{pmatrix} 0,5 \\ 0,5 \\ 0,5 \end{pmatrix} \textcircled{1}$

b) GA: $d=10, e_{\min} = -3, e_{\max} = 4, l=2$

Problematische Stellen:

(i) $\underline{R}_{33} = -1999 = -0,1999 \cdot 10^4 \hat{=} -0,2 \cdot 10^4 = \underline{-2000}$

$\sim A \hat{=} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1000 & 1 \end{pmatrix}}_{=L} \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0,001 & 2 \\ 0 & 0 & -2000 \end{pmatrix}}_{=R}$

(ii) $\underline{\hat{y}}_3 = 1 - 1000 = -999 = -0,0999 \cdot 10^4 \hat{=} -0,1 \cdot 10^4 = \underline{-1000}$

$\rightarrow \underline{\hat{y}} = \begin{pmatrix} 0 \\ 1 \\ -1000 \end{pmatrix}$

(iii) $\underline{\hat{x}}_3 \hat{=} \frac{-1000}{-2000} = \underline{0,5}$

$\underline{\hat{x}}_2 \hat{=} 1000 \cdot (1 - 2 \cdot 0,5) = \underline{0} \rightarrow \underline{\hat{x}} \hat{=} \begin{pmatrix} 0,5 \\ 0 \\ 0,5 \end{pmatrix} \leftarrow !$

$\underline{\hat{x}}_1 \hat{=} 0 + 0,5 = \underline{0,5}$