Institute for Applied and Numerical Mathematics
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Numerical methods in mathematical finance

Winter semester 2018/19

Problem Sheet 5

Problem 13 (Itô's formula and integration by parts)
Let $\{W_t \colon t \geq 0\}$ be a Wiener process and suppose that

$$u\colon [0,T] \to \mathbb{R}$$

is continuously differentiable. Show that

$$\int_0^T u(s)\,\mathrm{d}W_s = u(T)W_T - \int_0^T W_s u'(s)\,\mathrm{d}s.$$

Hint: Introduce $F(t,x) := u(t)x$ and use Itô's formula.

Problem 14 (Moments of the GBM)
A stochastic process S_t is called a geometric Brownian motion (GBM) if it satisfies the SDE

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}W_t$$

with constants $\mu, \sigma \in \mathbb{R}$. For a fixed initial value S_0, the above SDE has the solution

$$S_t = S_0 \exp(at + \sigma W_t) \quad \text{with} \quad a = \mu - \frac{\sigma^2}{2}\,.$$

Show that S_t has the following properties:

(a) $\mathbb{E}(S_t) = S_0 e^{\mu t}$

(b) $\mathbb{E}(S_t^2) = S_0^2 e^{(2\mu+\sigma^2)t}$

(c) $\mathrm{Var}(S_t) = S_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right)$

The problems on this sheet will be discussed on **19th November, 2018**.

The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.