Institute for Applied and Numerical Mathematics
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Numerical methods in mathematical finance

Problem Sheet 6

Winter semester 2018/19

Problem 15 (Binomial Method)

In this problem we want to prove Lemma 4.3.1 of the lecture: Let $V(t,S)$ be the value of a European option with payoff function $\psi(S)$ and maturity $T>0$. Then, the binomial method yields the approximation

$$V_{00} = e^{-rT} \sum_{j=0}^{N} \binom{N}{j} p^j (1-p)^{N-j} \psi(S_{jN}) \,.$$

Problem 16 (Black-Scholes formula with dividends)

Assume dividends are paid at a constant rate δ continuously, with the underlying stock being modeled by the SDE

$$dS_t = (\mu - \delta) S_t dt + \sigma S_t dW_t.$$

Derive the corresponding Black-Scholes equation.

Hint: Use a replication strategy, and follow the steps from the derivation of the standard Black-Scholes equation (Section 3.2 in the Lecture). How can the equation for $\mathrm{d}V(t,S_t)$ be adapted to obtain a self-financing portfolio in the case that dividends are paid?

Programming Exercise 3 (Implementation of the binomial method)

In this problem we want to implement and test the Binomial Method from Chapter 4 of the lecture in MATLAB. Proceed as follows:

- Write a MATLAB function

  `value = binomialMethod(N, r, sigma, K, S0, T, option)`

  which approximates the value of an option with the Binomial Method. The parameter `option` declares the style of the option, i.e. European/American and Put/Call.

- Write a MATLAB function

  `value = BlackScholes(S,t,K,r,sigma,T,option)`

  which computes the exact value of an European put or call option via the Black-Scholes formula. **Hint:** Use the MATLAB-function `normcdf`.

- In order to test the functions, write a MATLAB script `p3_main.m` where you compute the value of one of the following options:
  - European Call option
  - American Call option
  - European Put option
  - American Put option

  The output in the command window should say something like "Binomial method with $N = 500$ yields the value 1.847".

  Choose reasonable values for r, σ, K, S_0, T and in particular for N. Use the Black-Scholes solution to verify your results for both the European style options.

  Make sure that all parameter values (including the `option` parameter) may be exchanged in a convenient way.

The problems on this sheet will be discussed on **26th November, 2018**.

Assistance with this programming exercise will be provided in the tutorials on **27th November, 2018**, **28th November, 2018** and **29th November, 2018**.

The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.