Numerical methods in mathematical finance
Winter semester 2018/19

Problem Sheet 8

Programming Exercise 4 (Convergence of the Euler-Maruyama method)
Here we examine the convergence of the Euler-Maruyama method. For a start, copy the MATLAB functions eulerMaruyama and wienerprocess from the second programming exercise to your current working directory. Then, create a script p4_main.m which should contain your solutions to part (a) and (d) below.

(a) Examine the strong order of convergence of the Euler-Maruyama method as follows:
• Generate 10000 Wiener paths with $T = 10$ and $N = 2^{12}$.
• Compute the exact solution of the GBM for each Wiener path using $S_0 = 1$, $\mu = 0.3$, $\sigma = 0.4$.
• Use the Euler-Maruyama method to compute approximations with $N = 2^k$ for $k = 4, \ldots, 12$ using the given Wiener paths.
  \textbf{Caution:} Choose the correct time points of the Wiener path!
• Compute the strong error for each approximation.
• Confirm the strong order of convergence by plotting the error versus step-size with the MATLAB-function loglog.

(b) Modify the function eulerMaruyama by adding a parameter option such that it is possible to eventually use the Milstein correction term. Then confirm the strong order of convergence for the Milstein-Runge-Kutta method.

(c) Modify your function eulerMaruyama such that it is possible to call it without passing a precalculated Wiener path. In this case calculate random Wiener increments in each time step. \textbf{Hint:} Use the MATLAB-variable nargin.

(d) Examine the weak order of convergence (with respect to $F(x) = x$) of the Euler-Maruyama method as follows:
• Use the parameters $T = 10$, $S_0 = 1$, $\mu = 0.3$ and $\sigma = 0.4$.
• Use the Euler-Maruyama method to compute approximations with $N = 2^k$ for $k = 4, \ldots, 10$ each with 500000 samples. (Do not precalculate Wiener paths.)
  \textbf{Caution:} Due to memory issues save only the expectation at each time step!
• Compute the weak error for each approximation and confirm the weak order of convergence in a loglog plot.

Problem 20 (Gronwall’s inequality)
Let $\alpha : [0, T] \to \mathbb{R}_+$ be a positive continuous function. Show that if there exist constants $a > 0$ and $b > 0$ such that
\[ a \leq \alpha(t) \leq a + b \int_0^t \alpha(s)ds \]
for all $t \in [0, T]$, then we have
\[ \alpha(t) \leq ae^{bt}. \]

Problem 21 (A weak order 2 method)
During the discussion of a weak order 2 method we encountered the double integral
\[ I_{(1,0)}(t) = \int_t^\infty \int_{t_n}^x dW(r)ds = \int_t^\infty W(s) - W(t_n)ds \]
and claimed the properties
\[ \mathbb{E}(I_{(1,0)}(t)) = 0, \quad \mathbb{E}(I_{(1,0)}^2(t)) = \frac{(t-t_n)^3}{3} \]
and
\[ \mathbb{E}(I_{(1,0)}(t)[W(t) - W(t_n)]) = \frac{(t-t_n)^2}{2}. \]
Verify these properties.

The problems on this sheet will be discussed on 10\textsuperscript{th} December, 2018.
The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.