Institute for Applied and Numerical Mathematics
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Numerical methods in mathematical finance

Winter semester 2018/19

Problem Sheet 8

Programming Exercise 4 (Convergence of the Euler-Maruyama method)

Here we examine the convergence of the Euler-Maruyama method. For a start, copy the MATLAB functions `eulerMaruyama` and `wienerprocess` from the second programming exercise to your current working directory. Then, create a script `p4_main.m` which should contain your solutions to part (a) and (d) below.

(a) Examine the strong order of convergence of the Euler-Maruyama method as follows:

- Generate 10000 Wiener paths with `T = 10` and `N` = 2^{12}.
- Compute the exact solution of the GBM for each Wiener path using `S0 = 1`, μ `= 0.3`, σ `= 0.4`.
- Use the Euler-Maruyama method to compute approximations with `N` = 2^k for $k = 4, \dots, 12$ using the given Wiener paths.
  Caution: Choose the correct time points of the Wiener path!
- Compute the strong error for each approximation.
- Confirm the strong order of convergence by plotting the error versus step-size with the MATLAB-function `loglog`.

(b) Modifiy the function `eulerMaruyama` by adding a parameter `option` such that it is possible to eventually use the Milstein correction term. Then confirm the strong order of convergence for the Milstein-Runge-Kutta method.

(c) Modify your function `eulerMaruyama` such that it is possible to call it without passing a precalculated Wiener path. In this case calculate random Wiener increments in each time step. **Hint:** Use the MATLAB-variable `nargin`.

(d) Examine the weak order of convergence (with respect to $F(x) = x$) of the Euler-Maruyama method as follows:

- Use the parameters `T = 10`, `S0 = 1`, μ `= 0.3` and σ `= 0.4`.
- Use the Euler-Maruyama method to compute approximations with `N` = 2^k for $k = 4, \dots, 10$ each with 500000 samples. (Do not precalculate Wiener paths.)
  Caution: Due to memory issues save only the expectation at each time step!
- Compute the weak error for each approximation and confirm the weak order of convergence in a `loglog` plot.

Problem 20 (Gronwall's inequality)

Let $\alpha : [0,T] \to \mathbb{R}_+$ be a positive continuous function. Show that if there exist constants $a > 0$ and $b > 0$ such that

$$a \le \alpha(t) \le a + b \int_0^t \alpha(s)ds$$

for all $t \in [0,T]$, then we have

$$\alpha(t) \le ae^{bt}.$$

Problem 21 (A weak order 2 method)

During the discussion of a weak order 2 method we encountered the double integral

$$\mathcal{I}_{(1,0)}(t) = \int_{t_n}^t \int_{t_n}^s \mathrm{d}W(r)\mathrm{d}s = \int_{t_n}^t W(s) - W(t_n)\mathrm{d}s$$

and claimed the properties

$$\mathbb{E}(\mathcal{I}_{(1,0)}(t)) = 0, \qquad \mathbb{E}(\mathcal{I}^2_{(1,0)}(t)) = \frac{(t-t_n)^3}{3}$$

and

$$\mathbb{E}(\mathcal{I}_{(1,0)}(t)[W(t) - W(t_n)]) = \frac{(t-t_n)^2}{2}.$$

Verify these properties.

The problems on this sheet will be discussed on **10th December, 2018**.

The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.