Institute for Applied and Numerical Mathematics
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Numerical methods in mathematical finance

Winter semester 2018/19

Problem Sheet 10

Problem 24 (Polar method)

The polar method is an improvement of the Box-Muller method. In this problem we show that the implied transformation of uniformly distributed random variables on the unit disc yields uniformly distributed random variables on the unit square.

Let $B := \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 < 1\}$ denote the unit disc and $S := (0,1) \times (0,1)$ the unit square. Consider a random variable $V = (V_1, V_2)$, which is uniformly distributed on the unit disc B with density

$$f(x) = \begin{cases} \frac{1}{\pi}\,, & x \in B, \\ 0\,, & \text{else.} \end{cases}$$

Let $\phi\colon B \to S$ be given by

$$\phi(v_1, v_2) = \begin{pmatrix} v_1^2 + v_2^2 \\ H(v_1, v_2) \end{pmatrix}, \qquad (v_1, v_2) \in B,$$

with the function H from the last problem sheet. Show that

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} := \phi(V_1, V_2)$$

is uniformly distributed on the unit square $S = (0,1) \times (0,1)$.

Programming Exercise 5 (Monte-Carlo method)

In this problem we want to approximate the value of a European option using the Monte-Carlo method. We assume that the stock price is modeled by the geometric Brownian motion (GBM). Proceed as follows:

- Write a MATLAB function

  ```
  value = MCEuropean(m,S,K,r,sigma,T,option,N)
  ```

  which approximates the value of an option using the Monte-Carlo method. The parameter `option` declares the style of the option, i.e. Put or Call. The parameter `N` should be optional: If no `N` is passed to the function, use the exact solution formula for the GBM to compute the stock prices. Otherwise, the parameter `N` determines the number of time-steps for the Euler-Maruyama approximation of the GBM.

- In order to test the method, write a script `p5_main.m` where you approximate the value of a European option for reasonable parameter values. Observe the convergence of the Monte-Carlo method by

  (i) choosing a large sample size ($m \approx 500.000$) and varying the number of time-steps.

  (ii) choosing a large number of time-steps and varying the sample size.

  Use the MATLAB function `BlackScholes` from Programming Exercise 3 to obtain the exact option value.

  Caution: Due to computational time issues it may be advisable to use the MATLAB function `MCEuropean` without passing N in (ii).

The problems on this sheet will be discussed on **7th January, 2019**.

The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.