Institute for Applied and Numerical Mathematics
Prof. Dr. Tobias Jahnke • M. Sc. Benny Stein

Numerical methods in mathematical finance

Winter semester 2018/19

Problem Sheet 11

Problem 25 (Discrepancy)

Let $R \subseteq [0,1]^d$ be an arbitrary axially parallel d-dimensional rectangle in the unit cube $[0,1]^d$. The discrepancy of the point set $P = \{x^1, \dots, x^m\}$ is

$$D_m := \sup_R \left| \frac{A(P,R)}{m} - \mathrm{vol}(R) \right|, \quad A(P,R) = \#(P \cap R),$$

with the *star discrepancy* $D_m^\star$ defined in the same way but maximizing the difference over those rectangles $R^\star$ which are anchored in the origin, i.e., $R^\star = \bigtimes_{i=1}^d [0, y_i)$ for some $y \in \mathbb{R}^d$.

(a) Show that the discrepancies D_m and $D_m^\star$ satisfy

$$D_m^\star \leq D_m \leq 2^d D_m^\star.$$

(b) Show that we have the lower bounds

$$D_m \geq \frac{1}{m} \quad \text{and} \quad D_m^* \geq \frac{1}{2^d m}.$$

(c) Let $d = 1$. Show that the sequence

$$x^j = \frac{2j-1}{2m} \quad \text{for} \quad j = 1, \dots, m$$

has $D_m^\star = 1/(2m)$. Compare this with the second estimate in part (b).

Problem 26 (Difference quotients)

Let $y \in C^4([a,b])$. Choose $1 < m \in \mathbb{N}$, let $h = (b-a)/m$ and $x_k = a + kh$ for $k = 0, \dots, m$. Show that the second derivative of y can be approximated as follows:

$$\max_{k=1,\dots,m-1} \left| y''(x_k) - \frac{y(x_k+h) - 2y(x_k) + y(x_k-h)}{h^2} \right| \leq \frac{h^2}{12} \left\| \frac{\mathrm{d}^4 y}{\mathrm{d}x^4} \right\|_\infty$$

The problems on this sheet will be discussed on **14th January, 2019**.

The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.