Institute for Applied and Numerical Mathematics
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Numerical methods in mathematical finance

Winter semester 2018/19

Problem Sheet 13

Problem 28 (A-stability)
Show that the implicit Euler method and the trapezoidal rule are A-stable.

Problem 29 (Finite Differences and Black Scholes – Derivation of the discrete problem)
In this problem, we examine the application of the finite difference method to a European capped symmetric power call with payoff

$$V(T,S) = \min\left\{L, \left((S-K)^+\right)^p\right\} \tag{1}$$

and $T, K, p, L > 0$. The value $V(t,S)$ of this option evolves according to the Black-Scholes equation

$$\partial_t V(t,S) + \frac{\sigma^2}{2} S^2 \partial_S^2 V(t,S) + rS\partial_s V(t,S) - rV(t,S) = 0\,, \quad S \in (0,\infty)\,,\ t \in [0,T]$$

with $\sigma, r > 0$ and terminal condition (1); cf. Section 7.5 from the lecture notes.

As a first step, we understand the discretization process of the Black-Scholes equation. In the next problem (Programming Exercise 7), we implement the numerical method. (Therefore you will need the solution to this problem to do Programming Exercise 7.)

(a) Reproduce the truncation of the domain, the time inversion and the homogenization of the Black-Scholes equation from the lecture.

(b) Perform the "space discretization" of the PDE from (a) using the finite difference method. Choose $m+1$ discretization points ($m-1$ inner points) and carry out all details of this discretization. Write down the resulting ODE system

$$w_h'(t) = M_h w_h(t)$$

together with the initial value. Give an explicit formula for the matrix M_h.

(c) Recall the time discretization of the ODE system from (b) using the implicit Euler method and the trapezoidal rule, respectively. Write down the linear system which has to be solved in each time-step.

Programming Exercise 7 (Finite Differences and Black Scholes – Implementation)
In this programming exercise, we want to approximate the value of a European capped symmetric power call. To this end, we implement the numerical method from Problem 29 in MATLAB. Proceed as follows:

(a) Write a MATLAB function

```
Mh = blackScholesFD(sigma,r,h,Svec),
```

which assembles the matrix $M_h \in \mathbb{R}^{(m-1)\times(m-1)}$ of the ODE system. The vector `Svec` corresponds to $(S_1, \dots, S_{m-1})^T \in \mathbb{R}^{(m-1)}$ and contains the inner points of the "space discretization".

(b) Write a MATLAB script `p7_main.m` in which you approximate the value of a European capped symmetric power call via finite differences using the implicit Euler method and the trapezoidal rule, respectively. Choose reasonable values for all parameters and show your results in a 3d plot.

All remaining problems and programming exercises will be discussed in the problem classes on 4th February, 2019 and 5th February, 2019.

Do not forget to submit your solutions to the programming exercises 2–7.

In order to submit an exercise send an email containing your name and matriculation number to `benny.stein@kit.edu` <u>and</u> `uycyp@student.kit.edu`. All MATLAB files should be included via an attachment.

If you submit several exercises in one email, make sure to put each exercise in a seperate folder (and then compress the folder which contains all of the exercises afterwards). Add the subject line "Submission NumMethMathFin".

Check that you really added the attachments.

The submission deadline for all programming exercises is 17th February, 2019.

The problems on this sheet will be discussed on **4th February, 2019**.

The link http://www.math.kit.edu/ianm3/edu/nummethmathfin2018w/en/ leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the problem class.