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Splitting Methods, Exercise sheet 1

Exercise 1:

Compute the adjoint method of

$a)$ the explicit Euler method.

$b)$ the imlicit midpointrule.

$c)$ the Lie-Splitting.

$d)$ the Strang-Splitting.

Exercise 2:

Prove that the Lie splitting method applied to the initial value problem

$$y'(t) = f(y(t)) = f^{[1]}(y(t)) + f^{[2]}(y(t)), \quad 0 < t < T, \quad y(0) = y_0 \tag{1}$$

with $f : \mathbb{R} \to \mathbb{R}$ and $f^{[i]} : \mathbb{R} \to \mathbb{R}$, $i = 1, 2$ is consistent of order 1. In particular state the regularity assumptions on f and $f^{[i]}$, $i = 1, 2$. Furthermore precisely state the error structure (i.e., under which conditions on $f^{[i]}$, $i = 1, 2$ do we obtain consistency of order $p > 1$?).

Exercise 3:

Prove that the Strang splitting method applied to (1) is consistent of order 2 and conclude a lemma for this result (see Lemma 1.1). In particular state the regularity assumptions on f and $f^{[i]}$, $i = 1, 2$. Furthermore precisely state the error structure (i.e., under which conditions on $f^{[i]}$, $i = 1, 2$ do we obtain consistency of order $p > 2$?).

Exercise 4:

Consider the following model problem:

$$y'(t) = Ly(t) = Ay(t) + By(t), \quad y(0) = y_0 \in \mathbb{R}^n, \quad t \in [0, T],$$

with matrices $L, A, B \in \mathbb{R}^{n \times n}$.

Prove that the local error of the Lie splitting method is bounded by

$$\left\| y(h) - e^{hA} e^{hB} y_0 \right\| \leq Ch^2 \left\| [A, B] \right\|.$$

Will be discussed in the exercise class on: 05.11.2013.