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Splitting Methods, Exercise sheet 3

Exercise1:

Consider a general splitting method

$$\Psi_h = \varphi^{[2]}_{b_m h} \circ \varphi^{[1]}_{a_m h} \circ \cdots \circ \varphi^{[2]}_{b_1 h} \circ \varphi^{[1]}_{a_1 h}.$$

Conclude a condition for the coefficients a_i, b_i with $i = 1, \ldots, m$ which is necessary for the consistency of Ψ_h.

Reminder: A method Ψ_h is consistent if the local error fulfills the following property

$$\lim_{h \to 0} \frac{y(h) - \Psi_h}{h} = 0.$$

The exact solution is denoted by y and the timestep by h.

Exercise2:

Let $\varphi^{[i]}_{t_i}$ be the flow of the differential equation

$$y'(t) = f^{[i]}(y(t)), \quad \text{for} \quad i = 1, \ldots, m.$$

Show that

$$(\varphi^{[m]}_{t_m} \circ \cdots \circ \varphi^{[1]}_{t_1})(y_0) = \exp(t_1 D_1) \cdots \exp(t_m D_m) \mathrm{Id}(y0).$$

Here the D_i with $i = 1, \ldots, m$ are the corresponding Lie-derivatives.

Exercise3:

Let $f^{[1]}(y)$ and $f^{[2]}(y)$ be obtained on an open set. Prove that the corresponding flows $\varphi^{[1]}_s$ and $\varphi^{[2]}_t$ commute if and only if $[D_1, D_2] = 0$.

Exercise4:

Construct a splitting method of order 3 by using the order conditions.

Will be discussed in the exercise class on: 03.12.2013.