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## Splitting Methods, Blatt 4

**Exercise 1:**

We define a general splitting method with $m$-stages by

$$\Psi_h = \varphi^{[2]}_{b_m h} \circ \varphi^{[1]}_{a_m h} \circ \cdots \circ \varphi^{[2]}_{b_1 h} \circ \varphi^{[1]}_{a_1 h},$$

where the coeffients $a_i, b_i \in \mathbb{C}$ with $i = 1, \ldots, m$. Moreover, we can define these method by the following recursion

$$\Psi^j_h = \varphi^{[2]}_{b_j h} \circ \varphi^{[1]}_{a_j h} \circ \Psi^{j-1}_h \quad \text{with} \quad \Psi^0_h = \mathrm{Id}.$$

Prove that the methods $\Psi^j_h$ can be formally written as

$$\Psi^j_h = \exp(c^1_{1,j} h E^1_1 + c^1_{2,j} h E^1_2 + c^2_{1,j} h^2 E^2_1 + \mathcal{O}(h^3))\mathrm{Id},$$

with

$$E^1_1 = D_1, \quad E^1_2 = D_2, \quad E^2_1 = \tfrac{1}{2}[D_1, D_2],$$

and

$$\begin{aligned} c^1_{1,j} &= c^1_{1,j-1} + a_j, \\ c^1_{2,j} &= c^1_{2,j-1} + b_j, \\ c^2_{1,j} &= c^2_{1,j-1} + a_j b_j + c^1_{1,j-1} b_j - c^2_{2,j-1} a_j. \end{aligned}$$

**Exercise 2:**

Let $\varphi, \Psi : \mathbb{R}^{2d} \to \mathbb{R}^{2d}$ be symplectic. Prove that the composition $\varphi \circ \Psi$ is also symplectic.

**Exercise 3:**

Show that the implicit midpoint rule (as defined in the lecture) is symplectic.

**Exercise 4:**

We define a general composition method with $m$-stages by

$$\zeta_h = \Phi_{\beta_m h} \circ \Phi^*_{\alpha_m h} \circ \cdots \circ \Phi_{\beta_1 h} \circ \Phi^*_{\alpha_1 h},$$

where the coeffients $\alpha_i, \beta_i \in \mathbb{C}$ with $i = 1, \ldots, m$. $\Phi$ denotes a first-order method and $\Phi^*$ is its adjoint (also a first-order method). Moreover, we can define these method by the following recursion

$$\zeta^j_h = \Phi_{\beta_j h} \circ \Phi^*_{\alpha_j h} \circ \zeta^{j-1}_h \quad \text{with} \quad \zeta^0_h = \mathrm{Id}.$$

We assume also the following relations

$$\begin{aligned} \Phi_h &= \exp(hC_1 + h^2 C_2 + h^3 C_3 + \ldots)\mathrm{Id}, \\ \Phi^*_h &= \exp(hC_1 - h^2 C_2 + h^3 C_3 - \ldots)\mathrm{Id}. \end{aligned}$$

Prove that the methods $\zeta^j_h$ can be formally written as

$$\zeta^j_h = \exp(\gamma^1_{1,j} h E^1_1 + \gamma^2_{1,j} h^2 E^2_1 + \mathcal{O}(h^3))\mathrm{Id},$$

with $E^k_1 = C_k$ and

$$\begin{aligned} \gamma^1_{1,j} &= \gamma^1_{1,j-1} + \alpha_j + \beta_j, \\ \gamma^2_{1,j} &= \gamma^2_{1,j-1} + \alpha^2_j - \beta^2_j. \end{aligned}$$

**Will be discussed in the exercise class on:** 17.12.2013.