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SM · WS 13/14
14.01.2014

Splitting Methods, Blatt 5

Exercise 1:

We consider a Hamiltonian of the form

$$H(q,p) = T(p) + V(q), \quad q,p \in \mathbb{R}^d,$$

where T represents the kinetic energy and V is the potential energy. The equations of motion are then given by

$$\begin{aligned} q' &= \frac{\partial}{\partial p} T(p) \\ p' &= -\frac{\partial}{\partial q} V(q). \end{aligned} \tag{1}$$

First we consider the following Hamiltonian $H(q,p) = \frac{1}{2}(p^2 + q^2)$ (Harmonic oscillator).

a) Conclude the equations of motion explicitly.

b) Implement the Euler and symplectic Euler method.

c) Use these time integrators to compute approximations of (1) and check numerically in how far the energy is conserved. Choose $(p_0, q_0) = (0, 4)$ as the initial data and the timestep $h = \frac{1}{10}$.

d) Compute also the exact solution and plot the results.

e) Show that the modified Hamiltonian is given by

$$\widetilde{H}(p,q,h) = \frac{2\arcsin(\frac{h}{2})}{h\sqrt{4-h^2}}(p^2 + hpq + q^2)$$

and check its conservation numerically.

Exercise 2:

In this exercise we consider the following Hamiltonian $H(q,p) = \frac{1}{2}(p_1^2 + p_2^2) - \frac{1}{\sqrt{q_1^2+q_2^2}}$ (Kepler problem).

a) Conclude the equations of motion explicitly.

b) Use the Euler and the symplectic Euler method to compute a simulation of the 2-body problem. Choose the initial values

$$q_1(0) = 0.4, \quad q_2(0) = 0, \quad p_1(0) = 0, \quad p_2(0) = 2$$

and the timestep $h = \frac{1}{50}$.

c*) Implement the Störmer-Verlet (Leap-frog) method and the method of Heun (a special Runge-Kutta-Scheme). Repeat part b) by using the initial data

$$q_1(0) = 0.8, \quad q_2(0) = 0, \quad p_1(0) = 0, \quad p_2(0) = \sqrt{\frac{3}{2}}.$$

Choose a small timestep and compare the energy conservation.

Part c*) is an optional part of this exercise.

References:

[1] Blanes, Casas, Murua:
Splitting and Composition Methods in the Numerical Integration of Differential equations,
Bol. Soc. Esp. Mat. Apl. 45 (2008) 89–145.

[2] Hairer, Lubich, Wanner:
Geometric Numerical Integration, Structure-Preserving Algorithms for Ordinary Differential Equations,
Second Edition, Springer 2006.

Will be discussed in the exercise class on: 21.01.2014.