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Splitting Methods, Exercise sheet 6

Exercise1:

Interpret the leapfrog method (störmer verlet scheme) as a numerical time-integrator for second order odes as a splitting method.

Reference:

[1] E. Hairer, C. Lubich, G. Wanner,
Geometric numerical integration illustrated by the Störmer-Verlet method
Acta Numerica 2003, pp. 399-450.

Exercise2:

Consider the linear Schrödinger equation

$$\begin{aligned} &i\partial_t u = -\Delta u + V(x)u, \quad x \in \mathbb{T} = (-\pi, \pi) \\ &u(0) = \cos(x) \end{aligned}$$

with periodic boundary conditions and $V(x) = \cos(x)$.

a) Implement the Strang splitting method which is defined by

$$u_n = \left(e^{-i\frac{\tau}{2}V(x)} e^{i\Delta\tau} e^{-i\frac{\tau}{2}V(x)}\right)^n u_0 \approx e^{-i(-\Delta+V)n\tau} u_0 = u(t_n)$$

as a numerical time-integrator. Use a pseudo-spectral method to discretize the problem in space. Compute therefore the exact solution of

$$i\partial_t u = -\Delta u$$

in Fourier space.

b) Compute a reference solution at $T = 1$ and check the order of this splitting method numerically.

Will be discussed in the exercise class on: 11.02.2014.