Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825

Institut für Angewandte
und Numerische Mathematik

Prof. Dr. A. Rieder
Haojun Li, MSc.
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Wavelets - Theory and Applications

Problem Set 12

36. Show that the Gaußian $\varphi(t) = \mathrm{e}^{-t^2/2}$ is not a scaling function.

37. A scaling function φ is called *interpolating scaling function* if $\varphi(0) = 1$ and $\varphi(m) = 0$ for $m \in \mathbb{Z}\backslash\{0\}$.
Show that the symbol H of an interpolating scaling function satisfies

$$H(\omega) + H(\omega + \pi) = 1.$$

38. Prove the following properties of cardinal B-splines $B_n = B_{n-1} \star \chi_{[0,1]}$ with $B_1 = \chi_{[0,1]}$:
    (a) $B_n(t) = \int_{t-1}^{t} B_{n-1}(x)\mathrm{d}x$,
    (b) $B_n'(t) = B_{n-1}(t) - B_{n-1}(t-1)$,
    (c) $B_n(t) = \frac{t}{n-1} B_{n-1}(t) + \frac{n-t}{n-1} B_{n-1}(t-1)$,
    (d) to any $k \in \mathbb{Z}$ there is a $p_k \in \Pi_{n-1}$ such that $B_n|_{[k,k+1]} = p_k|_{[k,k+1]}$,
    (e) $B_n(t) = 2^{1-n} \sum_{k=0}^{n} \binom{n}{k} B_n(2t-k)$,
    (f) $\sum_{k\in\mathbb{Z}} B_n(t-k) = 1$,
    (g) $\int_{\mathbb{R}} B_n(t)\mathrm{d}t = 1$,
    (h) $\int_{\mathbb{R}} B_n(t)B_n(t-x)\mathrm{d}t = B_{2n}(n+x)$ (use that $B_n \star B_m = B_{n+m}$ and $B_n(x) = B_n(n-x)$),
    (i) $\sum_{k\in\mathbb{Z}} |\widehat{B}_m(\omega + 2\pi k)|^2 = \sum_{\ell=-m+1}^{m-1} B_{2m}(m+\ell)\mathrm{e}^{-\imath\ell\omega}$.