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Wavelets
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Problem Sheet 11 of January 20, 2014

Exercise 32:
Let φ be an orthogonal scaling function, that is, $\langle \varphi, \varphi_{0,k} \rangle_{L^2(\mathbb{R})} = \delta_{0,k}$, and let h be its filter. We define a corresponding wavelet by

$$\psi = \sum_{k \in \mathbb{Z}} g_k \varphi_{1,k} \quad \text{with } g_k = (-1)^{1-k} h_{2l+1-k}$$

for any choice of $l \in \mathbb{Z}$. Show:

(a) $\sum_{n \in \mathbb{Z}} h_n h_{n-2k} = \delta_{0,k}$,

(b) $\langle \psi_{0,m}, \psi_{0,k} \rangle_{L^2(\mathbb{R})} = \delta_{m,k}$.

Exercise 33: (Shannon's sampling theorem)
Let $f \in L^2(\mathbb{R})$ be b-bandlimited, that is, $\widehat{f}(\omega) = 0$ for almost all $\omega \in \mathbb{R} \setminus [-b, b]$, where $b > 0$. Show that

$$f(\cdot) = \sum_{k \in \mathbb{Z}} f(\delta k) \operatorname{sinc}\Big(\frac{\pi}{\delta}(\cdot - \delta k)\Big)$$

for any positive $\delta \leq \pi/b$. Hence, a bandlimited signal is completely determined by its discrete samples.
Hint: Expand $\widehat{f}$ in a Fourier series in $L^2(-\pi/\delta, \pi/\delta)$ and use $\widehat{\operatorname{sinc}} = \sqrt{\pi/2}\chi_{[-1,1]}$.

Exercise 34:
Let h be a filter associated with a scaling function φ. Prove: If $H(\omega)$ has a zero of order p at π, then $\widehat{\varphi}^{(l)}(2k\pi) = 0$ for any $k \in \mathbb{Z} \setminus \{0\}$ and any $l < p$.

These excersises are discussed in the problem class on **Thursday, January 23, 2014**.