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Wavelets
Winter Semester 2013/2014

Problem Sheet 13 of February 03, 2014

Exercise 38:
Let ψ be an orthogonal wavelet associated with the orthogonal scaling function φ. Let ϱ denote a 2π-periodic function satisfying $|\varrho(\omega)| = 1$, $\omega \in \mathbb{R}$. Show that $\widetilde{\psi}$ defined by

$$\widehat{\widetilde{\psi}} = \varrho\,\widehat{\psi}$$

is also an orthogonal wavelet associated with φ.

Exercise 39:
Let $\{h_k\}_{0\le k\le 2N-1}$ be the scaling coefficients of the Daubechies scaling function of order N. Let $g_k = (-1)^k h_{2N-1-k}$, $k = 0, \dots, 2N-1$, be the corresponding wavelet coefficients. Verify that

$$\sum_{k=0}^{2N-1} k^m g_k = 0, \quad m = 0, \dots, N-1\,.$$

Exercise 40:
Let the orthogonal scaling function φ and the associated orthogonal wavelet ψ satisfy

$$\int_{\mathbb{R}} x^k \psi(x)\,\mathrm{d}x = 0\,, \quad k = 0, \dots, M-1\,,$$

and

$$\int_{\mathbb{R}} \varphi(x)\,\mathrm{d}x = 1\,, \quad \int_{\mathbb{R}} x^k \varphi(x)\,\mathrm{d}x = 0\,, \quad k = 1, \dots, M-1\,.$$

Moreover, we assume that ψ and φ have compact support. Show that the symbol H has the following representations:

(a) $H(\omega) = \left(\dfrac{1+\mathrm{e}^{-i\omega}}{2}\right)^M L(\omega)\,,$

(b) $H(\omega) = 1 + \left(1-\mathrm{e}^{-i\omega}\right)^M \widetilde{L}(\omega)\,.$

Herein, L and $\widetilde{L}$ are trigonometric polynomials.

These excersises are discussed in the problem class on **Thursday, February 06, 2014**.