Karlsruhe Institute of Technology

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Wavelets
Winter Semester 2013/2014

Problem Sheet 14 of February 10, 2014

Exercise 41:
Let DW_N be the Daubechies wavelet of order N. Verify:

$$\int_{\mathbb{R}} t^m \, \mathrm{DW}_N(t)\,\mathrm{d}t = 0\,, \quad m = 0, \ldots, N-1\,.$$

Exercise 42:
Let DS_N be the Daubechies scaling function of order N. Show that, for any $m \in \{0, \ldots, N-1\}$,

$$x^m = \sum_{k\in\mathbb{Z}} c_{m,k} \mathrm{DS}_N(x-k) \text{ pointwise for almost all } x \in \mathbb{R}\,,$$

where

$$c_{m,k} = \int_{\mathbb{R}} (t+k)^m \mathrm{DS}_N(t)\,\mathrm{d}t\,.$$

Exercise 43:
Let DS_N be the Daubechies scaling function of order N. Show: For every $i = 0, \ldots, N-1$ there exists a polynomial $p_i \in \Pi_i$ such that

$$p_i = \sum_{k\in\mathbb{Z}} k^i \mathrm{DS}_N(\,\cdot - k) \quad \text{a.e. in } \mathbb{R}\,.$$

These excersises are discussed in the problem class on **Thursday, February 13, 2014**.