

Wavelets
Winter Semester 2013/2014
Problem Sheet 4 of November 18, 2013

Exercise 12:

Let $\psi = \sum_{k \in \mathbb{Z}} a_k \chi_{[k, k+1[}$ with $\sum_{k \in \mathbb{Z}} |k| |a_k| < \infty$ and $\sum_{k \in \mathbb{Z}} a_k = 0$.

(a) Show that ψ is a wavelet.

Hint: Consider Lemma 2.5 with $\beta = 1$.

(b) Which additional restrictions on $\{a_k\}_{k \in \mathbb{Z}}$ guarantee order 1, 2 and 3?

Exercise 13:

Let ψ be a wavelet of order $N \in \mathbb{N}$ with compact support $[T_1, T_2]$. Define

$$\varrho(x) = \frac{1}{(N-1)!} \int_{T_1}^x (x-z)^{N-1} \psi(z) \, dz.$$

Show that $\text{supp } \varrho = [T_1, T_2]$ and

$$\frac{d^N}{dx^N} \varrho = \psi.$$

These excersises are discussed in the problem class on **Thursday, November 21, 2013**.