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Wavelets
Winter Semester 2013/2014

Problem Sheet 5 of November 25, 2013

Exercise 14:
Let $f\colon \mathbb{R}\to\mathbb{R}$ be Hölder continuous of non-integer order $\alpha>0$ about $x\in\mathbb{R}$, that is, for all $t\in\mathbb{R}$

$$|f(t)-p_x(t)| \le \mathrm{K}\,|x-t|^\alpha\,,$$

where p_x is a polynomial of degree $\lfloor\alpha\rfloor$. Show that p_x is uniquely given.

Exercise 15:
Let $\psi\in L^1(\mathbb{R})$ be a wavelet and $f\in L^2(\mathbb{R})\cap L^\infty(\mathbb{R})$. Show that for almost every $b\in\mathbb{R}$

$$|W_\psi f(a,b)| \le \|W_\psi f(a,\cdot)\|_{L^\infty} = O\big(|a|^{1/2}\big) \quad \text{as } a\to 0\,.$$

Hint: Use Young's inequality:

$$\|u*v\|_{L^p} \le \|u\|_{L^1}\|v\|_{L^p}\,,\quad u\in L^1(\mathbb{R})\,,\, v\in L^p(\mathbb{R})\,,\, 1\le p\le\infty\,.$$

Exercise 16:
Let $\psi\in L^2(\mathbb{R})$ be an even and real-valued function with

$$0 < c_\psi = \sqrt{2\pi}\int_0^\infty \frac{\widehat{\psi}(\omega)}{\omega}\,\mathrm{d}\omega < \infty\,.$$

Then, for $f\in L^1(\mathbb{R})\cap L^2(\mathbb{R})$ we have that

$$f(b) = c_\psi^{-1}\int_0^\infty \langle f, T^b D^a\psi\rangle_{L^2(\mathbb{R})} a^{-3/2}\,\mathrm{d}a\,.$$

These excersises are discussed in the problem class on **Thursday, November 28, 2013**.