Karlsruhe Institute of Technology

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Wavelets
Winter Semester 2013/2014

Problem Sheet 7 of December 09, 2013

Exercise 20:
Let $\{\varphi_k\}_{k\in\Gamma}$ and $\{\widetilde{\varphi}_k\}_{k\in\Gamma}$ be two normalized frames in H, $\|\varphi_k\|_H = \|\widetilde{\varphi}_k\|_H = 1$, that are dual to each other. Show: If $0 < A \leq B$ are the frame bounds of $\{\varphi_k\}_{k\in\Gamma}$, then

$$A \leq 1 \leq B\,.$$

Hint: Use the statement of Exercise 19.

Exercise 21:
Let $a_0 > 1$ and let $\psi \in L^2(\mathbb{R})$ satisfy

$$|\widehat{\psi}(\omega)| \leq C\,|\omega|^\alpha(1+|\omega|)^{-\gamma}\,,$$

where $C, \alpha > 0$ and $\gamma > 1+\alpha$. Show that:

(a) $\Psi(\omega) = \sum_{m\in\mathbb{Z}} |\widehat{\psi}(a_0^m\omega)|^2$ is bounded in $\omega \in \mathbb{R}$.

(b) $\beta(s) = \sup_{|\omega|\in[1,a_0]} \sum_{m\in\mathbb{Z}} |\widehat{\psi}(a_0^m\omega)||\widehat{\psi}(a_0^m\omega + s)|$ decays as fast as $(1+|s|)^{-(\gamma-\alpha)}$ for $|s| \to \infty$.

Hint: Split up the series over $\mathbb{Z}$ into two series over negative and positive integers.

Exercise 22:
Let (ψ, a_0, b_0) be a wavelet frame with frame operator $F\colon L^2(\mathbb{R}) \to \ell^2(\mathbb{Z}^2)$. Show that $(F^*F)^{-1}$ commutes with dilations by a_0^l, $l \in \mathbb{Z}$, that is,

$$(F^*F)^{-1}D^{a_0^l} = D^{a_0^l}(F^*F)^{-1}\,.$$

These excersises are discussed in the problem class on **Thursday, December 12, 2013**.