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Splitting Methods — Exercise Sheet 10 January 22, 2015

On this exercise sheet we want to get more familiar with symplectic mappings.

For $y = (p, q) \in \mathbb{R}^{2d}$ we consider a Hamiltonian system

$$\dot{y} = J^{-1}\nabla H(y), \quad y(0) = (p_0, q_0) \in \mathbb{R}^{2d}, \qquad J = \begin{bmatrix} 0 & I_d \\ -I_d & 0 \end{bmatrix}. \tag{1}$$

Exercise 20: (Symplectic Euler method and Störmer-Verlet method)

Let $H(p, q) = T(p) + P(q)$, $H \in C^2(\mathbb{R}^{2d}; \mathbb{R})$ the Hamiltonian for system (1).

a) Show that the symplectic Euler method Φ_{SE}^{τ} is indeed a symplectic mapping, i.e. show that the mapping $(p_n, q_n) \mapsto (p_{n+1}, q_{n+1}) = \Phi_{SE}^{\tau}(p_n, q_n)$ is symplectic, where

$$p_{n+1} = p_n - \tau \frac{\partial P}{\partial q}(q_n)$$
$$q_{n+1} = q_n + \tau \frac{\partial T}{\partial p}(p_{n+1})$$

b) Show that the Störmer-Verlet method is symplectic.

Discussion in the problem class wednesday 3:45 pm, in room 1C-03 in building Allianzgebäude 5.20.

♣ : Please try to do exercises marked with ♣ at home.