Karlsruher Institut für Technologie

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Aspects of Numerical Time Integration — Exercise Sheet 1 April 11, 2014

Our aim in the first exercises is to learn how to implement simple time integrators for ordinary differential equations of the type $y'(t) = Ay(t)$ with a matrix $A \in \mathbb{R}^{n\times n}$ in MATLAB.

In order to get used to the techniques we start with the explicit and implicit Euler method applied to a second order differential equation.

Exercise 1: (the explicit Euler method)

Consider the second order differential equation

$$z''(t) + \omega^2 z(t) = 0, \quad z(0) = z_0,\ z'(0) = 0. \tag{1}$$

a) What is the exact solution $z(t)$ of this equation?

b) Rewrite this differential equation as a first order system of the form $y' = Ay$, $y(0) = y_0 \in \mathbb{R}^2$, $A \in \mathbb{R}^{2\times 2}$.

Now set $\omega^2 = \frac{3}{2}$, $z_0 = 3$.

(c) write a MATLAB function for the explicit Euler method and solve the first order system numerically with time step size $h_1 = 0.1$.

(d) plot the numerical and the exact solution on the time interval $[0, 20]$ in one figure. What do you observe?

(e) how does the numerical solution change if you use step sizes $h_2 = 0.05$, $h_3 = 0.025$ and $h_4 = 0.001$ instead?

Exercise 2: (the implicit Euler method)

a) Repeat steps (c), (d) and (e) of Exercise 1 but considering the implicit Euler method instead of the explicit Euler method.

b) plot the numerical solutions of the explicit and implicit Euler method with time step size $h = 0.1$ together with the exact solution into one figure. Can you explain the behaviour of the numerical solutions?

Exercise 3: (phase plot)

Create a so called *phase plot* for the numerical solutions of (1) obtained by the explicit and implicit Euler method.

Therefore plot $y(t)$ against $y'(t)$.

Again, what can you observe?

Discussion in the exercises tuesday 3:45 pm, the 15.04.2014 in room K2 (Kronenstraße 32).