

Aspects of numerical time integration — Exercise Sheet 2

April 22, 2014

Our aim in the second exercise is to gain knowledge on how to create order plots for various numerical methods.

An order plot is useful to visualize the numerical order of a method by plotting the error of the numerical solution against the step sizes by which we obtained the numerical solution.

Furthermore we want to apply the Lie and Strang splitting method to an ordinary differential equation.

For $\omega, z_0 \in \mathbb{R}$ We again consider the second order differential equation

$$z''(t) + \omega^2 z(t) = 0, \quad z(0) = z_0, \quad z'(0) = 0. \quad (1)$$

with exact solution $z(t) = z_0 \cos(\omega t)$ or equivalent the first order system for $y(t) := [z(t), z'(t)]^T$

$$y'(t) = Ay(t), \quad y(0) = [z_0, 0]^T, \quad A = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}. \quad (2)$$

In the following we set $\omega^2 = \frac{3}{2}$ and $z_0 = 3$.

Exercise 4: (order plot)

Create an order plot for the explicit and implicit Euler method applied to the first order system (2).

Therefore proceed as follows:

- set an initial time step size h_0 (for example $h_0 = 2^{-10}$).
- solve the equation on the time interval $[0, T]$ for some $T > 0$ with the numerical method using the time step size h_j .
- compute the maximal error $r_j := \max_n |y_{n,j} - y(t_n)|$ of the numerical solution $y_{n,j}$ with respect to the exact solution $y(t_n)$.
- repeat steps b) and c) M times with step size $h_{j+1} = 2 \cdot h_j, j = 0, \dots, M - 1$ instead.
- create a log-log-Plot with X-Data $h_{vec} = [h_0, \dots, h_M]$ and Y-Data $r_{vec} = [r_0, \dots, r_M]$.

Do this procedure a) - e) for both the explicit and implicit Euler method on the time interval $[0, 5]$. Start with $h_0 = 2^{-10}$ and increase the step size up to $h = 2^{-5}$ ($M = 6$ Iterations).

- Add the lines h and h^2 in the plot of step e).
What can you see?

Exercise 5: (the Lie and Strang splitting method)

Consider once more the system (2) with $\omega^2 = \frac{3}{2}$, $z_0 = 3$ but now decompose A by $A = B + C$, where

$$B := \begin{bmatrix} 0 & 0.5 \\ -1 & 0 \end{bmatrix}, \quad C := \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}.$$

The resulting first order system

$$y'(t) = By(t) + Cy(t), \quad y(0) = y_0 \quad (3)$$

is equivalent to (1). Therefore its exact solution is given by $y(t) = [z(t), z'(t)]^T$.

- write a MATLAB function to solve (3) using Lie splitting
- write a MATLAB function to solve (3) using Strang splitting
- which numerical order of these splitting methods do you expect?
- create an order plot for the Lie and Strang splitting method (in one figure) applied to (3) on the time interval $[0, 5]$. What do you observe?

Discussion in the exercises tuesday 3:45 pm, on April 22, 2014 in room K2 (Kronenstraße 32).