Karlsruher Institut für Technologie

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Aspects of numerical time integration — Exercise Sheet 5 May 15, 2014

On this exercise sheet we want to consider the linear Schrödinger equation with some potential $\Phi\colon \mathbb{R} \to \mathbb{R}$, i.e.

$$\partial_t u = i\Delta u + i\Phi(x)u, \qquad u(0,x) = g(x), \qquad x \in \mathbb{T}, \quad t \in [0,T], \tag{1}$$

on the torus $\mathbb{T}$. For implementation reasons we assume periodic boundary conditions (recall that this is necessary to apply spectral methods).

We set $\mathbb{T} := [-40\pi, 40\pi]$ and $T := 10$. Furthermore we choose $g(x) = e^{-x^2/2}$.
Discretize space with $N = 2048$ grid points and choose a time step size of $\tau = 0.001$.

Exercise 10:

First we want to consider equation (1) with $\Phi = 0$.

a) Write a MATLAB function to solve the linear Schrödinger equation

$$\partial_t u = i\Delta u, \qquad u(0,x) = g(x), \qquad x \in \mathbb{T}, \quad t \in [0,T],$$

using spectral methods as for the transport equation in exercise 9.

b) Plot the result in an animated plot using a `for` loop (compare exercises 8 and 9). But note that the solution is complex such that you have to plot its real part and its imaginary part seperately. You can do this very elegant by using subplots.

Now consider problem (1) with $\Phi(x) := e^{\sin(2x)} + \cos(x)$.

(c) Use a Strang splitting method to solve problem (1) numerically by considering the subproblems

$$\begin{aligned} \partial_t u &= i\Delta u =: Au, \qquad & u(0,x) &= u_0(x), \\ \partial_t u &= i\Phi(x)u =: Bu, \qquad & u(0,x) &= v_0(x). \end{aligned}$$

As a reminder:
The Strang splitting method is given by

$$u(\tau, x) = e^{\frac{\tau}{2}A} e^{\tau B} e^{\frac{\tau}{2}A} u(0,x).$$

(d) Plot your results as in b).

Discussion in the exercises tuesday 3:45 pm, the 20.05.2014 in room K2 (Kronenstraße 32).