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Aspects of numerical time integration — Exercise Sheet 6   May 28, 2014

On this exercise sheet we want to consider the nonlinear Schrödinger equation (NLS)

$$\partial_t u = i\Delta u + i|u|^2 u, \qquad u(0,x) = g(x), \qquad x \in \mathbb{T}, \quad t \in [0,T], \tag{1}$$

on the torus $\mathbb{T}$. For implementation reasons we assume periodic boundary conditions (recall that this is necessary to apply spectral methods).

We set $\mathbb{T} := [-20\pi, 20\pi]$ and $T := 10$.
Discretize space with $N = 2048$ grid points and choose a time step size of $\tau = 0.001$.

Exercise 11:

First we want to consider so called soliton solutions of the NLS. Soliton solutions are solutions which do not change their shape over time. An (exact) soliton solution of the NLS (1) is given by

$$s(t,x) := \sqrt{2}\,\text{sech}(x)e^{it} = \frac{\sqrt{2}}{\cosh(x)}e^{it},$$

where $\cosh(x) = \frac{e^x + e^{-x}}{2}$. Split problem (1) into the two subproblems

$$\partial_t u = i\Delta u, \qquad u(0,x) = u_0(x), \tag{I}$$

$$\partial_t u = i|u|^2 u, \qquad u(0,x) = v_0(x). \tag{II}$$

a) Check that $s(t,x)$ indeed solves the NLS. Hint: Write the derivatives $\partial_t s$ and $\partial_{xx} s$ in terms of s.

b) Show that subproblem (II) can be solved exactly w.r.t. time t.

Now let $g(x) = s(0,x)$.

c) Write a MATLAB function to solve the NLS on the torus $\mathbb{T}$ applying Lie splitting.

d) Write a MATLAB function to solve the NLS on the torus $\mathbb{T}$ applying Strang splitting.

e) Compute a numerical solution with these MATLAB functions and plot your numerical results and the exact solution in an animated plot using subplots for real and imaginary part of the solution.

Now let $h(x) = \frac{\sqrt{2}}{2}\left(1 - \tanh(x - \pi)\cdot\tanh(x + \pi)\right)$.

f) Compute the numerical solution choosing the initial value $u(0,x) = h(x)$ and plot (only) the numerical results as in e). What happens with the shape of the solution over time? Does it keep the shape of the initial value?

Discussion in the exercises tuesday, the 03.06.2014 at 3:45 pm in room K2 (Kronenstraße 32).