

Aspects of numerical time integration — Exercise Sheet 7

June 10, 2014

On this exercise sheet we want to analyze the L^2 norm and energy (Hamiltonian) conservation of the numerical solutions obtained with the Lie and Strang splitting method applied to the nonlinear Schrödinger equation (NLS)

$$\partial_t u = i\Delta u + i|u|^2 u, \quad u(0, x) = g(x), \quad x \in \mathbb{T}, \quad t \in [0, T], \quad (1)$$

on the torus \mathbb{T} over long times. For implementation reasons we assume periodic boundary conditions. The Hamiltonian is given by

$$H(u, \bar{u}) = \int_{\mathbb{T}} |\nabla u|^2 + \frac{1}{2} |u|^4 dx.$$

We set $\mathbb{T} := [-20\pi, 20\pi]$ and $T := 5$.

Discretize space with $N = 2048$ grid points and choose a time step size of $\tau = 0.001$ such that $t_n = n\tau$, $n = 0, 1, 2, \dots$

Exercise 12: (L^2 norm conservation and conservation of the Hamiltonian)

As in exercise 11 we consider the initial value

$$g(x) = \frac{\sqrt{2}}{\cosh(x)}.$$

- a) Given the parameters from above, compute a numerical solution $u_n \approx u(t_n)$ of the NLS using Lie and Strang splitting. You can use the MATLAB functions from exercise 11 c) and d). Measure the CPU time that MATLAB needs to compute the solutions using the `tic` and `toc` commands.

IMPORTANT NOTE: For reasons of storage do not save each computed time step. Since we are interested in the L^2 norm and Hamiltonian in exercise 12 b) you can set up your code with two different approaches:

- (i) If you are only interested in some properties: after each iteration only compute and save the properties of the solution you are interested in (for example Hamiltonian or L^2 norm).
- (ii) If you also want to plot the solution after the computation: Save the initial value and then skip for example 20 time steps until you save again. You can do this by using the following command sequence. Here we round the values to make sure that we obtain integer numbers.

```
dtplot=20*dt; % skip 20 steps until next saving
plotgap=round(dtplot/dt);
dt=dtplot/plotgap;
nplots=round(T/dtplot); % compute the number of steps to save
```

- b) For each time step t_n now compute the corresponding L^2 norm and Hamiltonian of the solution u_n and save each in an array of the length of the number of time steps that you saved.
- c) Now plot the error $|||u_n||_{L^2} - ||u_0||_{L^2}|$ of the L^2 norm of u_n at each time step t_n compared to the L^2 norm of the initial value in a semilogarithmic plot. Use the command `semilogy(t_array, u_array)`.
- d) Do the same for the error of the Hamiltonian $|H(u_n, \bar{u}_n) - H(u_0, \bar{u}_0)|$

Exercise 13:

Repeat exercise 12, but now compute the solution on the time interval $t \in [0, 100]$ instead.

Here the skipping of some time steps before saving again is very important, because your RAM storage usually cannot hold that much data and your computer may freeze.

Discussion in the exercises tuesday, the 10.06.2014 at 3:45 pm in room K2 (Kronenstraße 32).