Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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56	57	58	59	60	$\sum$

Karlsruhe, January 26, 2010

Student Nr.:

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Worksheet No.12
Advanced Mathematics I

Exercise 56: Consider the real-valued functions $f(x) = x^2,\ g(x) = x$, $x \in [0,1]$. Verify the mean value theorem of integral calculus

$$\int_0^1 f(x)\,g(x)\,dx \;=\; f(\xi)\int_0^1 g(x)\,dx$$

by specifying $\xi \in (0,1)$ with the asserted property.

Exercise 57: Using integration by parts, evaluate the following definite integrals,

$$\text{(a)}\ \int_0^1 t\cdot\arctan(t)\,dt, \qquad \text{(b)}\ \int_0^{\frac{\pi}{2}} \cos^3(t)\cdot\cos(t)\,dt.$$

Also use integration by parts to evaluate the following indefinite integrals,

$$\text{(c)}\ \int \sin^2(x)\,dx, \qquad \text{(d)}\ \int x^2\cdot\ln(x)dx.$$

Exercise 58: Determine the indefinite integral

$$\int \frac{2}{\tan(\frac{x}{2})+\cos(x)-\sin(x)}dx$$

using the substitution $u = \tan(\frac{x}{2})$.

Exercise 59: Evaluate the integral by means of partial fraction decomposition:

$$\int \frac{x^3+6x^2+3x+18}{x^3+x^2+4x+4}\,dx.$$

Exercise 60: A pool of water is supplied through four feed pipes. At time $t = 0$ the pool contains 100 liters of water. Afterwards, $\frac{\ln 2}{2^t}$ liters of water per second [l/sec] flow through pipe number one into (or out of) the pool for $0 \le t \le 1$. During the same time via the second pipe there stream $9te^{-3t} + \frac{2}{1+t^2}$ l/sec. For $0 \le t \le 4$ via the third pipe pour $\frac{-1}{4\sqrt{t}}$ l/sec. Via the last pipe we have a stream of $\frac{1}{t^2}$ l/sec for $t \ge c \ge 4$. Determine the amount of water in the pool for $t = 2c$ sec.

Due date: Please hand in your homework on Thursday, February 4, 12:00, into the AM1-box near Seminar room 1C-03, Allianz-Gebäude (05.20).

Tutorial 12
Advanced Mathematics I

Exercise T45: Compute the following antiderivatives using partial integration

(a) $\int x^2 \sin x dx$, (b) $\int \arctan \frac{1}{x-1} dx$, (c) $\int (\ln y)^2 dy$.

(d) Show moreover that the following equation holds: $\int_0^{2\pi} \cos^2 x dx = \int_0^{2\pi} \sin^2 x dx = \pi$.

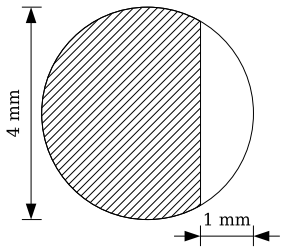
Exercise T46: Calculate the following integrals using suitable substitutions:

(a) $\int_0^{\frac{\pi}{2}} \cos(x) \cdot e^{\sin(x)}\, dx$, (b) $\int \frac{2x+7}{x^2+7x+3}\, dx$, (c) $\int \frac{\cos(\ln(x))}{x}\, dx$, (d) $\int \frac{1+\ln(x)}{x - x\ln(x)}\, dx$.

Exercise T47: Compute the following definite integral by means of partial fraction decomposition,

$$\int_0^{\frac{1}{2}} \frac{(x-2)\, dx}{x^3 + 2x^2 - x - 2} \quad .$$

Exercise T48:
A 30 cm long shaft of brass has as cross-section a circle with diameter 4 mm, which is flattened at one side by 1 mm (see sketch). Brass has the density 8.4 g/cm^3. How heavy is the shaft?



Tutorial date: Tuesday, February 2, 2010, 3:45-5:15 pm.