

1	2	3	4	5	$\Sigma$

## Worksheet No.1 Advanced Mathematics I

**Exercise 1:** Let  $A, B \subseteq \mathbb{R}$  be defined as

$$A := \{x \in \mathbb{R} : |x^2 - 2| \leq 4 - x\} \quad \text{and} \quad B := \{x \in \mathbb{R} : 1 - |x - 2| < \frac{1}{2}|x - 3|\}.$$

Give a representation of  $A \cup B$ ,  $A \cap B$  and  $A \setminus B$  as intervals.

**Exercise 2:** Evaluate the following sums:

$$(a) \sum_{n=7}^{42} \left(\frac{1}{3}\right)^n, \quad (b) \sum_{m=-1}^8 (n+1)^3, \quad (c) \sum_{\mu=0}^1 \sum_{\nu=2}^4 \frac{1}{\mu + \nu^2}.$$

**Exercise 3:**

(a) Evaluate the following sums:

$$(i) \sum_{k=0}^5 \binom{5}{k}, \quad (ii) \sum_{n=3}^5 \frac{\binom{n}{3}}{n!}.$$

(b) Prove that for  $n \geq m \geq r \geq 0$  it holds

$$\binom{n}{m} \cdot \binom{m}{r} = \binom{n}{r} \cdot \binom{n-r}{m-r}.$$

**Exercise 4:** Determine all solutions of the following system of linear equations:

$$(a) \begin{array}{rcl} -6x_1 & -9x_2 & +x_3 = -8 \\ -6x_1 & -7x_2 & -x_3 = -4 \end{array} \quad (b) \begin{array}{rcl} 5x_1 & +3x_2 & -2x_3 = 2 \\ -2x_1 & -2x_2 & +3x_3 = 0 \\ -8x_1 & -2x_2 & -5x_3 = -4 \end{array}$$

$$(c) \begin{array}{rcl} -3x_1 & +4x_2 & -3x_3 = -5 \\ 3x_1 & -2x_2 & +3x_3 = 7 \\ -2x_1 & +4x_2 & -2x_3 = -1 \end{array}$$

**Exercise 5:** Find all solutions of the linear system of equations

$$\begin{array}{rcl} 3x_1 & & +x_3 & -x_4 & = & 4 \\ -8x_1 & & +\alpha x_2 & -(\alpha+2)x_3 & +x_4 & = -8 \\ -8x_1 & -(2\alpha+6)x_2 & +(2\alpha+5)x_3 & -2x_4 & = & 2\beta \\ -4x_1 & -(2\alpha+2)x_2 & +2\alpha x_3 & +3x_4 & = & -1+\beta \end{array}$$

depending on the parameters  $\alpha, \beta \in \mathbb{R}$ .

## Tutorial 1 Advanced Mathematics I

**Exercise T1:** Determine the set of all  $x \in \mathbb{R}$ , that fulfill the following condition:

$$(a) \quad \frac{(x+1)(3-x)}{(x+5)^2} \leq 0, \quad (b) \quad |x|^3 = x^3 + 2x^2 - 3x.$$

**Exercise T2:** Evaluate the following sums:

$$(a) \quad \sum_{n=17}^{63} n, \quad (b) \quad \sum_{n=1}^8 (n-1/2)^2, \quad (c) \quad \sum_{\nu=1}^4 \sum_{k=1}^{\nu} \nu(\nu-k).$$

**Exercise T3:**

(a) Evaluate the following sums:

$$(i) \quad \sum_{n=-1}^1 \binom{5}{4}, \quad (ii) \quad \sum_{n=4}^7 \binom{n}{4}.$$

(b) Prove that for  $n \geq r \geq 1$  it holds

$$\binom{n}{r} = \frac{n}{r} \cdot \binom{n-1}{r-1}.$$

**Exercise T4:**

(a) Determine the solution of the system of linear equations

$$\begin{array}{rccccrcr} x_1 & - & x_2 & + & x_3 & + & x_4 & = & 0 \\ 2x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & 0 \\ 3x_1 & + & 2x_2 & + & x_3 & & & = & 3 \end{array}$$

(b) For which real numbers  $\alpha$  and  $\beta$  does the following system of linear equations have (i) a unique solution, (ii) more than one solution and (iii) no solution?

$$\begin{array}{rcl} \alpha x_1 + x_2 + 2x_3 & = & 1 \\ -x_1 + 3x_2 + x_3 & = & \beta \\ 2x_1 + \quad \quad 2x_3 & = & 2 \end{array}$$

For detailed information regarding this course please check the page  
<http://www.mathematik.uni-karlsruhe.de/iag1/lehre/am12009w/en>

**Tutorial date:** Tuesday, October 27, 2009, 3:45-5:15 pm.