Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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Karlsruhe, November 3, 2009

Student Nr.:

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Worksheet No.2
Advanced Mathematics I

Exercise 6: Prove by induction on $n \in \mathbb{N}$ that

(a) $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}$, (b) $\displaystyle\sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)} = \frac{n}{3n+1}$.

Exercise 7: Prove by induction over $n \in \mathbb{Z}_{\geq 0}$:

$$\sum_{k=0}^{2n} i^k k = \begin{cases} n(1-i), & n \text{ even} \\ -(n+1)+ni, & n \text{ odd} \end{cases}$$

Exercise 8: Given the complex numbers $z_1 = 1+i$, $z_2 = 2-3i$, $z_3 = \sqrt{3}+i$, compute

(a) the real and the imaginary part of the numbers $\overline{z_j}$, $-z_j$, $z_j\overline{z_j}$, $\dfrac{1}{z_j}$, $z_j - \overline{z_j}$ and $|z_j|$, $j = 1, 2$, as well as

$$\frac{z_1}{z_1+z_2}, \qquad \text{and} \qquad z_1^3 z_2^2.$$

(b) z_3's representation in polar coordinates (r, φ), where φ represents the principal value of the argument of z_3.

Exercise 9: Sketch the set of all complex numbers z that satisfy each of the following conditions:

(a) $|\mathrm{Re} z| + |\mathrm{Im} z| \leq 4$,

(b) $|z|^2 \leq 2\mathrm{Re} z$,

(c) $z^4 + (2i+2)z^2 + 4i = 0$.

Exercise 10: Compute the real and imaginary parts of the number $z = (1+i)^5$

(a) with the aid of the binomial theorem,

(b) by expressing $1+i$ in polar coordinates.

(c) Which of the above methods should you use to compute $w = (1-i)^{17}$?
Determine $\mathrm{Re}\, w$ and $\mathrm{Im}\, w$.

Due date: Please hand in your homework on Thursday, November 12, 12:00, into the AM1-box near Seminar room 1C-03, Allianz-Gebäude (05.20).

Tutorial 2
Advanced Mathematics I

Exercise T5: Use mathematical induction to show

(a) $\displaystyle\sum_{k=1}^{2^n} \frac{1}{k} \geq 1 + \frac{n}{2}, \quad n \in \mathbb{N}.$ (b) $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}, \quad n \geq 1.$

Exercise T6:

(a) Given the complex numbers

i) $z = 3 - i$ ii) $z = 3 + 4i$

plot each of the numbers $\overline{z}$, $-z$, $z\overline{z}$, $\frac{1}{z}$, $z - \overline{z}$, $|z|$ in the complex plane by resolving them into their real and imaginary parts.

(b) Compute the polar coordinates $(r, \operatorname{Arg} z)$ of the following numbers

i) $-\frac{1}{4} + \frac{1}{4}\sqrt{3}i,$ ii) $-1 - i.$

Exercise T7: Sketch the set of all complex numbers z such that

(a) $|3z - 1 + 2i| \leq 2,$

(b) $|z - z_0| = |z - z_1|$ for $z_0 = 1 - i$, $z_1 = 2 + i$.

Exercise T8: Determine real and imaginary parts of all solutions $w \in \mathbb{C}$ of the equations

(a) $w^2 = -5 + 12i,$ (b) $w^2 + 6iw - 6 = 4i.$

For detailed information regarding this course please check the page
`http://www.mathematik.uni-karlsruhe.de/iag1/lehre/am12009w/en`

Tutorial date: Tuesday, November 10, 2009, 3:45-5:15 pm.