Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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26	27	28	29	30	$\sum$

Karlsruhe, December 1, 2009

Student Nr.:

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Worksheet No.6
Advanced Mathematics I

Exercise 26: Calculate the limits

$$\text{(a)} \lim_{x\to x_0} \frac{\sqrt{x}-\sqrt[4]{x}}{x-1}, \quad \text{for } x_0 \in \{0+, 1, 16, \infty\}, \qquad \text{(b)} \lim_{x\to\infty} x^\alpha(\sqrt{x+1}-\sqrt{x}), \quad \text{for } \alpha \in \mathbb{R}.$$

Exercise 27: At which points $x \in \mathbb{R}$ are the following functions $f_j : \mathbb{R} \to \mathbb{R}$ continuous?

$$\text{(a)} \quad f_1(x) := \begin{cases} \dfrac{x^3+4x^2+x-6}{x^3-3x+2}, & x \in \mathbb{R} \setminus \{1, -2\} \\ 0, & x = 1, \\ -\dfrac{1}{3}, & x = -2, \end{cases} \qquad \text{(b)} \quad f_2(x) := \begin{cases} x, & x \in \mathbb{Z}, \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 28: Consider the set $M = \{z \in \mathbb{C} : |\text{Re}z|+|\text{Im}z| \leq 4\}$ and the function $f : \mathbb{C} \to \mathbb{R}$, where $f(z) = 2|z|$.

(a) Why must f attain a maximum and minimum in M?

(b) Determine the maximum and minimum of f on M.

Exercise 29: Show that for any positive constants a, b, c the equation

$$\frac{(a+b)x+a-b}{x^2-1} + \frac{c}{x-2} = 1$$

has solutions in the intervals $[-1, 1]$ and $[1, 2]$, respectively.

Exercise 30: Show that at any given time there are two antipodal points on the equator at which the temperature is exactly the same.

Hint: Suppose that the temperature on the equator is a continuous function $\tau : [0, 2\pi] \to \mathbb{R}$. Construct a function that represents the temperature difference between antipodal points, and see if you can apply the Intermediate Value Theorem.

Due date: Please hand in your homework on Thursday, December 10, 12:00, into the AM1-box near Seminar room 1C-03, Allianz-Gebäude (05.20).

Tutorial 6
Advanced Mathematics I

Exercise T21: Calculate the limits of functions, if they exist:

$$\lim_{x\to 0} \frac{5}{x^2-1}, \qquad \lim_{x\to 1} \frac{(x-1)^2}{x^2-1}, \qquad \lim_{x\to -1} \frac{(x-1)^2}{x^2-1}, \qquad \lim_{x\to 0+} \frac{x}{\sqrt{x+1}-1}.$$

Exercise T22: Is it possible in each case to choose values y_1, y_2 such that the given functions $f, g : \mathbb{R} \to \mathbb{R}$ become continuous? Calculate those values or prove that a solution is not possible.

(a) $$f(x) = \begin{cases} \dfrac{x^4 - 10x^2 + 9}{x^2 - 4x + 3} & \text{für } x \in \mathbb{R}\backslash\{1,3\} \\ y_1 & \text{für } x = 1 \\ y_2 & \text{für } x = 3 \end{cases}$$

(b) $$g(x) = \begin{cases} \dfrac{x^3 + 4x^2 + x - 6}{x^3 - 3x + 2} & \text{für } x \in \mathbb{R} \setminus \{1,-2\} \\ y_1 & \text{für } x = 1 \\ y_2 & \text{für } x = -2 \end{cases}$$

Exercise T23: Consider the set $M = \{z \in \mathbb{C} : |z| \leq 2\}$ and the function $f : \mathbb{C} \to \mathbb{R}$, where

$$f(z) = \mathrm{Re}\big((3+4i)z\big).$$

(a) Decide whether M is open, closed, or compact, respectively. Show that f has a maximum and a minimum on M.

(b) Determine the maximum and minimum of f on M.

Exercise T24: Show that the graphs of the functions

$$f_1(x) = \sqrt{x} \quad \text{und} \quad f_2(x) = (x^2-1)^2$$

have at least two intersections on their common domain $\mathbb{R}_{\geq 0}$.

Tutorial date: Tuesday, December 8, 2009, 3:45-5:15 pm.