Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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Karlsruhe, December 1, 2009

Student Nr.:

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Worksheet No.7
Advanced Mathematics I

Exercise 31: Examine the following series for convergence.

(a) $\left(\sum_{n=0}^{\infty}\left(\frac{3+4i}{6}\right)^n\right)$, (c) $\left(\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}\right)$,

(b) $\left(\sum_{n=1}^{\infty}\frac{n^2(n+1)^2}{n!}\right)$, (d) $\left(\sum_{n=8}^{\infty}\frac{n+7\sqrt{n}}{n^3-n}\right)$.

Exercise 32: Determine the convergence behaviour of the following series and calculate the first four partial sums s_1, s_2, s_3 and s_4, respectively.

(a) $\left(\sum_{\nu=1}^{\infty}\left(\frac{1}{\nu(\nu+1)}-\frac{4}{\nu}\right)\right)$, (b) $\left(\sum_{\nu=2}^{\infty}(-1)^\nu\frac{\sqrt{\nu^2-4}}{\nu^2}\right)$, (c) $\left(\sum_{\nu=1}^{\infty}(-1)^\nu\frac{2\nu+1}{\nu(\nu+1)}\right)$.

Exercise 33: Show that the series $\left(\sum_{k=0}^{\infty}(-1)^k\frac{k+1}{2^k}\right)$ converges absolutely. Next, prove the following representation of its partial sums s_n

$$s_n = \frac{1}{9}\left(4+(-1)^n\frac{3n+5}{2^n}\right), \quad n = 0, 1, 2, \ldots,$$

using mathematical induction and use the result for determination of the limit s of this series.

Exercise 34: Determine all $\alpha \in \mathbb{R}$ for which the series

$$\sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{\alpha-1}{\alpha+1}\right)^k$$

converges. Hint: Consider the power series $\sum_{k=1}^{\infty}\frac{1}{k}z^k$.

Exercise 35: „Oh, these holidays again...“, thinks Lucifer to himself, whose blood boils from all the joy and happiness around these days. Out of spite he wants to cross out all terms from our series, where numbers with halo appear (for him, these are all numbers with zeroes), and so hopes to bring chaos to the world unnoticed. However, you can robably unravel his devilry with the inharmonic series. Would it then still be divergent?

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}+\frac{1}{16}+\frac{1}{17}+\frac{1}{18}+\frac{1}{19}+\frac{1}{21}+\ldots$$

Hint: How many reciprocal j-digit numbers are preserved?

Due date: Please hand in your homework on Thursday, December 17, 12:00, into the AM1-box near Seminar room 1C-03, Allianz-Gebäude (05.20).

Tutorial 7
Advanced Mathematics I

Exercise T25: Assess the convergence of the series (a), (b) using the ratio test and the series (c), (d) using the root test:

$$\text{(a)}\ \left(\sum_{k=0}^{\infty}\frac{2^k\cdot 3^k}{k!\cdot(2k+1)}\right),\quad \text{(b)}\ \left(\sum_{k=0}^{\infty}\frac{k^k}{k!\cdot 3^k}\right),\quad \text{(c)}\ \left(\sum_{k=0}^{\infty}\left(\frac{9}{10}+\frac{1}{k}\right)^k\right),\quad \text{(d)}\ \left(\sum_{k=0}^{\infty}\frac{4^k(1+2k)^k(1-k)^k}{(3+k)^{2k}2^k}\right).$$

Exercise T26: Consider the series $\left(\sum_{n=0}^{\infty} a_n\right)$, where

$$a_n = \begin{cases} \dfrac{-1}{2^n}, & n \text{ even} \\ \dfrac{1}{4^n}, & n \text{ odd} \end{cases}$$

(a) Show the absolute convergence of the series using the comparison test.

(b) Verify that the root test shows the series to be absolutely convergent, while the ratio test is inconclusive.

(c) Determine the precise value of the series by choosing a suitable decomposition.

Exercise T27: Prove the convergence of the series $\left(\sum_{n=1}^{\infty}\frac{(-n)^n}{(n+1)^{n+1}}\right)$ by means of the Leibniz' test.

Hint: Apply the monotonicity of the sequence $\left(1+\frac{1}{n}\right)^n$.

Exercise T28: A „copy“ of the Tower of Babylon is prepared by piling the cubes W_n with edge legth of $\frac{1}{n}$ meter, where $n = 1, 2, 3, ...$ In this process the base area of the $n+1^{\text{st}}$ cube is placed centered on the roof area of the n^{th} cube.

(a) How high will the tower be?

(b) Can the tower be brushed by a finite amount of dye?

(c) Can the builders manage with finite amount of conctrete, assuming the cubes are complete of concrete?

Tutorial date: Tuesday, December 15, 2009, 3:45-5:15 pm.