Karlsruhe Institute of Technology
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Karlsruhe, December 22, 2009

Student Nr.:

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Worksheet No.9
Advanced Mathematics I

Exercise 41: The following series representation of the logarithmus function can be applied to evaluate it approximatively on a computer:

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}.$$

(a) For which $x \in \mathbb{R}$ is this expansion in a power series possible, i.e. for which x does the series converge?

(b) How many elements of the power series are sufficient to evaluate $\ln(1.5)$ with an accuracy of two decimal places? Calculate $\ln(1.5)$ with an accuracy of two decimal places by applying the series representation above!

Exercise 42:

(a) Show that for $\varphi \in \mathbb{R}$ and $n = 0, 1, 2, 3, \ldots$ the following equality holds

$$\cos\big((n+1)\,\varphi\big) + \cos\big((n-1)\,\varphi\big) = 2\,\cos\varphi\,\cos(n\,\varphi).$$

(b) Consider for $n = 0, 1, 2, \ldots$ the function $T_n : [-1, 1] \to \mathbb{R}$ given by

$$T_n(x) := \cos(n\,\arccos(x)).$$

Prove that

$$T_{n+1}(x) + T_{n-1}(x) = 2\,x\,T_n(x).$$

(c) Conclude that the function T_n is a polynomial of degree n at most.

(d) The function T_5 can be expanded at the expansion point $x_0 = -1/2$ in a power series. Which radius of convergence does this power series have?

Exercise 43: Find all solutions $z \in \mathbb{C}$ of the equation

$$\cosh z - \frac{1}{2}\,(1 - 8i)\,e^{-z} = 2 + 2i.$$

Solve the quadratic equation by completing the square.

Exercise 44: For each of the following equations, determine the set of solutions $z \in \mathbb{C}$:

$$\text{(a)}\quad \cos\overline{z} = \overline{\cos z} \qquad\qquad \text{(b)}\quad e^{i\overline{z}} = \overline{e^{iz}}$$

Exercise 45: Solve the complex equations

(a) $(\sinh(iz) + \cosh(iz))^2 + 2\sin(2z) = 0,$ (b) $\sinh(iz) + \cosh(iz))\sin(2z) = \sqrt{2}(i\sin(z) + \cos(z)).$

Solve the quadratic equation by completing the square.

We wish you a merry Christmas and a happy new year!

Due date: Please hand in your homework on Thursday, January 14, 12:00, into the AM1-box near Seminar room 1C-03, Allianz-Gebäude (05.20).

Tutorial 9
Advanced Mathematics I

Exercise T33:

(a) Determine all $z \in \mathbb{C}$, which satisfy the equation

$$\cos z = 4.$$

Use the exponential representation of the cosine function.

(b) Determine all complex numbers $z \in \mathbb{C}$ that satisfy the equation

$$\cosh(z) = -1\,.$$

Use the representation of cosh in terms of the exponential function.

Exercise T34: Solve the following equation for $z \in \mathbb{C}$:

$$-5\cos z + 7\,i\sin z = 1.$$

Exercise T35: Prove the following formulas for $z = x + iy$, x, $y \in \mathbb{R}$:

$$\begin{aligned} \sin z &= \sin x\, \cosh y + i\, \cos x\, \sinh y, \\ \cos z &= \cos x\, \cosh y - i\, \sin x\, \sinh y. \end{aligned}$$

Hint: Use Theorem 4.21.

Exercise T36: The power a^x is defined by $a^x := e^{x \cdot \ln a}$ for $a > 0$, $x \in \mathbb{R}$. Show that:

(a) $(a^x)^y = a^{xy}$ for $a, b > 0$.

(b) a^x is strictly monotonically increasing for $a > 1$, and strictly monotonically decreasing for $0 < a < 1$.

When $a = 10$ the inverse function of $f(x) = 10^x$ is $\log_{10} x$.

(c) How can the value $\log_{10} x$ be computed, using the function $\ln x$?

(d) Show that

$$\begin{aligned} \log_{10}(xy) &= \log_{10} x + \log_{10} y \quad , \quad x, y > 0\ , \\ \log_{10}(x^y) &= y \log_{10} x \quad , \quad x > 0,\ y \in \mathbb{R}\ . \end{aligned}$$

Tutorial date: Tuesday, January 12, 2010, 3:45-5:15 pm.