Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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Karlsruhe, January 11, 2011

Student Nr.:

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Worksheet No.10
Advanced Mathematics I

Exercise 46: Compute the derivatives of the following functions in their domains of definition:

(a) $f(x) = \dfrac{x}{x^2+1}$, (b) $g(x) = \left(\dfrac{x+1}{x-1}\right)^2$, (c) $h(x) = (\sqrt{x}+1)\left(\dfrac{1}{\sqrt{x}} - 1\right)$,

(d) $u(x) = e^x \cos^2 x(\cos x + 3\sin x)$, (e) $v(x) = \dfrac{\sin x}{\cos x + \sin x}$.

Exercise 47: Let $\alpha \in \mathbb{R}$. Consider the following function defined on $[0, \infty)$

$$f(x) = \begin{cases} x^\alpha \sin \dfrac{1}{x}, & x > 0 \\ 0, & x = 0 \end{cases}.$$

Investigate (depending on α) if f is continuous, differentiable or continuously differentiable.

Exercise 48:

(a) Give the interval $I \subset \mathbb{R}$, where the function $f(x) = \cosh x$ is strictly monotonic increasing and thus invertible.

(b) Apply the representation $\cosh x = \frac{1}{2}(e^x + e^{-x})$ to determine the inverse function $f^{-1}(x) = \operatorname{Arcosh}(x)$ and its derivative $(f^{-1})'(x)$.

Exercise 49: The inverse function of the tangent is the function

$$\arctan : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

(a) Compute the derivative of arctan using the formula for computing the derivative of an inverse.

(b) arctan can be expanded in a power series for $|x| < 1$:

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}.$$

Compute $(\arctan x)'$ by differentiating this power series.

Exercise 50: Consider the differential equation $y'(x) - 2y(x) = 0$ for $x \in \mathbb{R}$ as well as the initial value $y(0) = 3$.

(a) The solution y can be represented as a power series of the form $y(x) = \sum_{k=0}^{\infty} a_k x^k$. Plug this ansatz into the differential equation and find a recursion formula for the coefficients a_k.

(b) Determine a_0 using the initial value, find an explicit representation for the coefficients a_k and give the solution y in a closed form.

Due date: Please hand in your homework until Thursday, January 20, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 10
Advanced Mathematics 1

Exercise T28: Compute the derivatives of the following functions in their domains of definition:

(a) $f(x) = \cos^2 x \cdot \cos(x^2)$, (b) $f(x) = \ln\left(\dfrac{e^x}{1+e^x}\right)$,

(c) $f(x) = \dfrac{\cos^2 x}{1+\cot x} + \dfrac{\sin^2 x}{1+\tan x}$, (d) $f(x) = \sin x + \dfrac{1}{\sin x}$.

Exercise T29: Consider the function

$$f(x) = \begin{cases} e^x & x \le 0 \\ \cos(x) + x & 0 < x \end{cases} .$$

(a) Show that f is continuously differentiable arbitrarily many times at $x \neq 0$.

(b) Show that f is (once) continuously differentiable at $x = 0$.

(c) Is f twice continuously differentiable at $x = 0$?

Exercise T30: Consider the the following power series:

$$f(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}, \qquad g(x) = \sum_{k=0}^{\infty} (k+1)x^k .$$

Determine the domains, in which the derivatives of the functions can be determined by differentiating the corresponding power series, and compute these derivatives.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, January 12, 2011, 14:00-15:30.