Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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56	57	58	59	60	$\sum$

Karlsruhe, January 25, 2011

Student Nr.:

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Worksheet No.12
Advanced Mathematics I

Exercise 56: Calculate the following integrals using suitable substitutions:

(a) $\int_0^{\frac{\pi}{2}} \cos(x) \cdot e^{\sin(x)}\, dx$, (b) $\int \frac{2x+7}{x^2+7x+3}\, dx$, (c) $\int \frac{\cos(\ln(x))}{x}\, dx$, (d) $\int \frac{1+\ln(x)}{x - x\ln(x)}\, dx$.

Exercise 57: Compute the following antiderivatives using partial integration

(a) $\int x^2 \sin x dx$, (b) $\int \arctan \frac{1}{x-1} dx$, (c) $\int (\ln y)^2 dy$.

(d) Show moreover that the following equation holds: $\int\limits_0^{2\pi} \cos^2 x dx = \int\limits_0^{2\pi} \sin^2 x dx = \pi$.

Exercise 58: Determine the indefinite integral

$$\int \frac{5e^{3x} + e^{2x} + 3e^x + 1}{e^{3x} + e^x}\, dx\,,$$

using a suitable substitution and a subsequent decomposition into partial fractions.

Exercise 59: For $n \in \mathbb{N}_{>0}$ we define

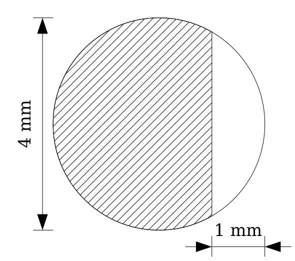
$$\Gamma(n) := \lim_{a\to\infty} \int\limits_0^a x^{n-1} e^{-x} dx.$$

Show that $\Gamma(n+1) = n \cdot \Gamma(n)$ and use this to prove $\Gamma(n+1) = n!$.

Remark: This integral exists more generally for all $n \in \mathbb{R} \setminus \{0, -1, -2, \ldots\}$ and defines a continuous function on this domain: the so-called *Gamma function*.

Exercise 60:

A 30 cm long shaft of brass has as cross-section a circle with diameter 4 mm, which is flattened at one side by 1 mm (see sketch). Brass has the density 8.4 g/cm^3. How heavy is the shaft?



Due date: Please hand in your homework until Thursday, **February 10**, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 12
Advanced Mathematics 1

Exercise T34: Compute the following integrals by the substitution method:

(a) $\displaystyle\int_1^4 \frac{(1+x)^2}{\sqrt{x}}dx, \quad u=\sqrt{x},$ (b) $\displaystyle\int_0^1 \frac{(2+t^2)t^3}{(1+t^2)^3}dt, \quad y=1+t^2,$ (c) $\displaystyle\int\limits_0^a a^r\,dr, \quad a>1, \quad w=r\ln a.$

Exercise T35: Compute the following integrals using partial integration:

(a) $\int x\cdot\sin(x)dx$ (b) $\int(x+2)e^{2x}dx$ (c) $\int x^2\cdot\ln(x)dx$
(d) $\int \frac{x}{\cos^2(x)}dx$ (e) $\int\sin^2(x)dx$ (f) $\int\cos^3(x)dx$

Exercise T36: Compute the following integrals using partial fraction decomposition:

$$\text{(a)} \int \frac{5x^2-11x+5}{x^3-4x^2+5x-2}\,dx, \qquad \text{(b)} \int \frac{x^3+6x^2+3x+18}{x^3+x^2+4x+4}\,dx.$$

Please note the following deadlines with respect to the **AM1-exam**:

- The exam takes place on 19 February 2011 from 8:00 until 10:00
- It is possible to register starting from 31 January 2011.
- The online registration has to be carried out by 15 February 2011. For later cancellations, please see the staff at the institute.

For detailed information please check the page
`www.math.kit.edu/iag1/lehre/hm1mach2010w/event/klausurhm12011w/en`

For further detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, January 26, 2010, 14:00-15:30.