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Student Nr.:

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Worksheet No.1 Advanced Mathematics I

Exercise 1: Let $A, B \subseteq \mathbb{R}$ be defined as

$$A := \{x \in \mathbb{R} : |x^2 - 2| \leq 4 - x\} \quad \text{and} \quad B := \{x \in \mathbb{R} : 1 - |x - 2| < \frac{1}{2} |x - 3|\}.$$

Give a representation of $A \cup B$, $A \cap B$ and $A \setminus B$ as intervals.

Exercise 2: Evaluate the following sums:

$$(a) \sum_{n=17}^{63} n, \quad (b) \sum_{n=1}^8 (n - 1/2)^2, \quad (c) \sum_{\nu=1}^4 \sum_{k=1}^{\nu} \nu(\nu - k).$$

Exercise 3:

(a) Evaluate the following sums:

$$(i) \sum_{n=-1}^1 \binom{5}{4}, \quad (ii) \sum_{n=4}^7 \binom{n}{4}.$$

(b) Prove that for $n \geq r \geq 1$ it holds

$$\binom{n}{r} = \frac{n}{r} \cdot \binom{n-1}{r-1}.$$

Exercise 4: Determine all solutions of the following systems of linear equations:

$$(a) \begin{array}{rcl} -6x_1 & -9x_2 & +x_3 = -8 \\ -6x_1 & -7x_2 & -x_3 = -4 \end{array} \quad (b) \begin{array}{rcl} 5x_1 & +3x_2 & -2x_3 = 2 \\ -2x_1 & -2x_2 & +3x_3 = 0 \\ -8x_1 & -2x_2 & -5x_3 = -4 \end{array}$$

$$(c) \begin{array}{rcl} -3x_1 & +4x_2 & -3x_3 = -5 \\ 3x_1 & -2x_2 & +3x_3 = 7 \\ -2x_1 & +4x_2 & -2x_3 = -1 \end{array}$$

Exercise 5:

(a) For which values of α and $\beta \in \mathbb{R}$ is the following linear system of equations

$$\begin{array}{rcl} 2x_1 & +3x_2 & & +1x_4 = \beta \\ 1x_1 & +1x_2 & & -1x_4 = 1 \\ & 2x_2 & +1x_3 & +3x_4 = 0 \\ 3x_1 & -1x_2 & +2x_3 & +\alpha x_4 = 2 \end{array}$$

solvable?

(b) Give the set of solutions \mathcal{L} of the linear system above and the set of solutions \mathcal{L}_0 of the appropriate homogeneous system.

Tutorial 1 Advanced Mathematics I

Exercise T1: Determine the set of all $x \in \mathbb{R}$, that fulfill the following condition:

$$(a) \quad \frac{(x+1)(3-x)}{(x+5)^2} \leq 0, \quad (b) \quad |x|^3 = x^3 + 2x^2 - 3x.$$

Exercise T2: Evaluate the following sums:

$$(a) \sum_{n=7}^{42} \left(\frac{1}{3}\right)^n, \quad (b) \sum_{n=3}^5 \frac{\binom{n}{3}}{n!}, \quad (c) \sum_{\mu=0}^1 \sum_{\nu=2}^4 \frac{1}{\mu + \nu^2}.$$

Exercise T3:

(a) Determine the solution of the system of linear equations

$$\begin{array}{rccccrcr} x_1 & - & x_2 & + & x_3 & + & x_4 & = & 0 \\ 2x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & 0 \\ 3x_1 & + & 2x_2 & + & x_3 & & & = & 3 \end{array}$$

(b) For which real numbers α and β does the following system of linear equations have (i) a unique solution, (ii) more than one solution and (iii) no solution?

$$\begin{array}{rcl} \alpha x_1 + x_2 + 2x_3 & = & 1 \\ -x_1 + 3x_2 + x_3 & = & \beta \\ 2x_1 + \quad \quad 2x_3 & = & 2 \end{array}$$

For detailed information regarding this course please check the page
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