Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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Karlsruhe, November 2, 2010

Student Nr.:

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Worksheet No.2
Advanced Mathematics I

Exercise 6: Prove the following formula by induction:

$$\sum_{j=1}^{n}(-1)^{j-1}j^2 \;=\; (-1)^{n-1}\binom{n+1}{2}, \quad n \in \mathbb{N}.$$

Exercise 7: Prove the following formulas by induction

$$\text{(a)}\sum_{k=1}^{n}\frac{1}{n+k} \geq \frac{1}{2}, \qquad \text{(b)}\sum_{l=0}^{n}\binom{n}{l} = 2^n.$$

Exercise 8: Determine all solutions of the linear systems

(a)
$$\begin{array}{rrrrcr} x_1 & +3x_2 & -x_3 & +x_4 & = & 1 \\ 2x_1 & +x_2 & +16x_3 & +x_4 & = & 11 \\ -x_1 & +2x_2 & & +2x_4 & = & 0 \\ & x_2 & +3x_3 & +x_4 & = & 2 \end{array}$$

(b)
$$\begin{array}{rcrcr} (1+i)x_1 & - & ix_2 & = & 2+3i \\ (2+i)x_1 & + & (3-i)x_2 & = & 4+7i \end{array}$$

Exercise 9: Determine all complex solutions z in the following equations:

(a)
$$z^2 + (2i+2)z + 4i = 0,$$

(b)
$$z^4 + (2i+2)z^2 + 4i = 0.$$

Exercise 10: Find all complex-valued solutions z of the equation

$$\frac{z-3}{z-i} + \frac{z-4+i}{z-1} = 2\,\frac{-3+2i}{z^2-(1+i)z+i}.$$

Hint: Simplify at first and then use the substitution $z = x + iy$, x, $y \in \mathbb{R}$.

Due date: Please hand in your homework until Thursday, November 11, 12:00 into the AM2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 2
Advanced Mathematics 2

Exercise T4: Use mathematical induction to show

(a) $\displaystyle\sum_{k=1}^{2^n} \frac{1}{k} \geq 1 + \frac{n}{2}, \quad (n \in \mathbb{N}_0)$ (b) $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}, \quad (n \in \mathbb{N}).$

Exercise T5: Find the values of $\alpha \in \mathbb{C}$, for which the solutions of the linear system of equations

$$\begin{array}{rrrcr} (i\alpha - i)z_1 & +(i\alpha^2 - i)z_2 & -(1+i)z_3 & = & -2-2i \\ (i-1)z_1 & +(i-1)z_2 & -iz_3 & = & -1-i \\ z_1 & +z_2 & -z_3 & = & -1 \end{array},$$

(a) don't exist;

(b) are an infinite set. In this case determine the set of solutions.

Exercise T6: Determine real and imaginary parts of all solutions $w \in \mathbb{C}$ of the equations

(a) $w^2 = -5 + 12i$, (b) $w^2 + 6iw - 6 = 4i$.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Friday, November 5, 2010, 11:30-13:00.