Karlsruhe Institute of Technology
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Karlsruhe, November 9, 2010

Student Nr.:

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Worksheet No.3
Advanced Mathematics I

Exercise 11: Given the complex numbers $z_1 = 1+i$, $z_2 = 2-3i$, $z_3 = \sqrt{3}+i$, compute

(a) the real and the imaginary part of the numbers $\overline{z_j}$, $-z_j$, $z_j\overline{z_j}$, $\dfrac{1}{z_j}$, $z_j - \overline{z_j}$ and $|z_j|$, $j = 1, 2$, as well as

$$\frac{z_1}{z_1+z_2}, \qquad \text{and} \qquad z_1^3 z_2^2.$$

(b) z_3's representation in polar coordinates (r, φ), where φ represents the principal value of the argument of z_3.

Exercise 12: Given the sequence $(a_n)_n$, $a_n = \dfrac{n+1}{(n+2)^2}$, $(n \in \mathbb{N})$, show that $a := \lim\limits_{n\to\infty} a_n = 0$, by specifying an index N_0 depending on a given $\varepsilon > 0$, such that the following is true:

$$|a_n - a| < \varepsilon \qquad \text{for all} \quad n \geq N_0 \ .$$

Give such an N_0 explicitly for i) $\varepsilon = \frac{1}{10}$, ii) $\varepsilon = \frac{1}{100}$, iii) $\varepsilon = 10^{-10}$.

Exercise 13: Let $n \in \mathbb{N}$. Compute the limits of the sequences

(a) $a_n = \dfrac{2011(1+n+n^2)}{n(n+2010)}$, (b) $b_n = \sqrt{n^2+an+b} - n$, $a, b \in \mathbb{R}$, n sufficiently big,

(c) $c_n = \left[1 + \left(-\dfrac{3}{5}\right)^n\right] \cdot \left[\dfrac{10^n}{n!} - \dfrac{3n^2+1}{(2n+1)^2}\right]$, (d) $d_n = \sqrt[n]{17 \cdot 2^n}\,\left(\sqrt{n+1} - \sqrt{n}\right)$.

Exercise 14: Determine the limits of the complex sequences:

$$\text{(a) } a_n = 2 + \frac{3}{4i\,n} + \left(\frac{1}{2} + \frac{1}{3}i\right)^n, \quad n \in \mathbb{N}, \qquad \text{(b) } b_n = \frac{(3i\,n+1)(2n+i)}{\sum\limits_{k=1}^{n} ik}, \quad n \in \mathbb{N}.$$

Exercise 15: Investigate if the following sequences converge. Compute the limit in case it exists.

$$(a) \ \ a_n = \left(\frac{n}{n+1} - \frac{n+1}{n}\right) \cdot \frac{n^2+3}{2n+2}, \quad n \in \mathbb{N}, \qquad (b) \ \ b_n = \left(2\frac{p_n}{n}\right)^n \ \text{ für } \ p_n = \prod_{k=1}^{n}\left(1+\frac{1}{k}\right), \quad n \in \mathbb{N}.$$

Hint for (b): Prove the equation $p_n = n+1$ by the mathematical induction, at first.

Due date: Please hand in your homework until Thursday, November 18, 12:00 into the AM1-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 3
Advanced Mathematics 1

Exercise T7:

(a) Given the complex numbers

$$\text{i) } z = 3 - i \qquad\qquad \text{ii) } z = 3 + 4i$$

plot each of the numbers $\overline{z}$, $-z$, $z\overline{z}$, $\frac{1}{z}$, $z - \overline{z}$, $|z|$ in the complex plane by resolving them into their real and imaginary parts.

(b) Compute the polar coordinates $(r, \operatorname{Arg} z)$ of the following numbers

$$\text{i) } -\frac{1}{4} + \frac{1}{4}\sqrt{3}i, \qquad \text{ii) } -1 - i.$$

Exercise T8: Consider the sequence $(a_n)_n$ with $a_n = \frac{n-1}{n+1}$, $n \in \mathbb{N}$. Determine an index N such that $|a_n - 1| < \varepsilon$ for every $n \geq N$, when

(a) $\varepsilon = \dfrac{1}{10}$, (b) $\varepsilon = \dfrac{1}{1000}$, (c) $\varepsilon > 0$ is arbitrary.

(d) Does the sequence $(a_n)_n$ converge? If so, to what limit?

Exercise T9: Determine the limit of the sequence $(a_n)_n$, where

$$\text{(a) } a_n = \sqrt{n^2 + n} - n \ , \qquad \text{(b) } a_n = \frac{n^4 - 2}{n^2 + 4} + \frac{n^3(3 - n^2)}{n^3 + 1}, \qquad \text{(c) } a_n = \frac{2 + i^n}{4 + n}, \ n \in \mathbb{N}.$$

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, November 10, 2010, 14:00-15:30.
Problem class date: Wednesday, November 10, 2010, 11:30-13:00.