Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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Karlsruhe, November 16, 2010

Student Nr.:

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Worksheet No.4
Advanced Mathematics I

Exercise 16: Determine all accumulation points of the sequences

$$\text{(a)} \quad a_n = \frac{1}{n} + 2(-1)^n, \quad n \in \mathbb{N}, \qquad \text{(b)} \quad b_n = \left(\frac{5n+7}{n}\right) i^n, \quad n \in \mathbb{N}.$$

Exercise 17: Analyze the following sequences for $n \in \mathbb{N}$ to determine whether they are bounded, monotonic, and convergent (in case of convergence you need not specify the limit).

$$(a) \quad a_n = \frac{1+n+n^2}{n(n+1)}, \qquad (b) \quad b_n = \frac{1+n+n^2}{n+1},$$

$$(c) \quad c_n = \frac{1}{1+(-2)^n}, \qquad (d) \quad d_n = \frac{1+(-2)^n}{1+2^n}.$$

Exercise 18: Let the sequence (a_n) be recursively defined by the formula

$$a_{n+1} = a_n^2 + \frac{1}{4}, \qquad n \in \mathbb{N} \cup \{0\}.$$

(a) Show that the sequence converges for every initial value $0 \leq a_0 \leq \frac{1}{2}$ and compute its limit.

(b) Show that the sequence diverges for each $a_0 > \frac{1}{2}$ (Hint: Show that $a_n \geq a_0 + nd$ where $d = (a_0 - 1/2)^2$).

(c) What happens when $a_0 < 0$?

Exercise 19: Given a constant $c \in \mathbb{R}$ we define the sequence $(z_n)_n$ by

$$z_0 = 0, \quad z_{n+1} = \frac{n^2 z_n^2 + c}{n+1}, \; n \in \mathbb{N}.$$

(a) Let $c = -2$, give a guess for a closed representation for $(z_n)_n$ for $n \geq 2$, prove this form, analyze for convergence and compute the limit.

(b) Show that the sequence is monotonously increasing for $c = 1$ and prove divergence by comparison with the sequence $v_n = (4/3)^{n-2}$.

Exercise 20: A student memorizes three pages of AM1 lecture notes a day. Overnight he forgets 4% of his total acquired knowledge. Assume that the lecture notes has infinite number of pages and that the student has no AM1 knowledge at his first semester day.

(a) Set up the sequence (in a recursive form) for the amount of knowledge w_n of the probant after expiration of n days and n nights.

(b) Prove that $(w_n)_n$ is monotonely increasing.

(c) Prove that $(w_n)_n$ is bounded by 75 pages from above.

(d) What will his level of knowledge be in the long run? (limit)

Due date: Please hand in your homework until Thursday, November 25, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 4
Advanced Mathematics 1

Exercise T10:

$a_n =$	1	n	$1/n$	$(-1)^n$	$(-1)^n n$	i^n/n	$(-1)^n n + n$	$a_{n-1}/2$, $a_0 = 1$
constant								
bounded								
unbounded								
convergent								
divergent								
improperly convergent								
monotonically decreasing						///		
strictly decreasing						///		
monotonically increasing						///		
strictly increasing						///		
alternating						///		
accumulation point(s)								
limit								

Assign the characteristics to different sequences $(a_n)_n$.

Exercise T11: Examine the following sequences for $n \in \mathbb{N}$ for boundedness, monotonicity and convergence. (Limits or accumulation points need not be specified). If appropriate, analyze suitable subsequences of each sequence.

(a) $a_n = \dfrac{1 + 6n + 2n^2}{(n+3)n}$,

(b) $b_n = 6 - \dfrac{6 + n^2}{n}$,

(c) $c_n = \dfrac{(-2)^{-n} + 1}{1 + 2n} - 1 + \dfrac{2n}{1 + 2n}$,

(d) $d_n = \dfrac{1 + 2^n}{1 + 2^n + (-2)^n}$.

Exercise T12: For $a \in \mathbb{R}_{>0}$ we define the sequence (x_n) in a recursive form by

$$x_{n+1} = 2x_n - a x_n^2, \quad n \in \mathbb{N} \cup \{0\}$$

for arbitrary $0 < x_0 < \frac{1}{a}$.

(a) Show $x_{n+1} \leq \frac{1}{a}$ for every $n \in \mathbb{N}_0$ (hint: completion of the square).

(b) Show by induction: $x_n \geq 0$ for every $n \in \mathbb{N}_0$ (hint: use part (a)).

(c) Why does $(x_n)_n$ converge? Determine $\lim\limits_{n\to\infty} x_n$.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, November 24, 2010, 14:00-15:30.