Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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21	22	23	24	25	$\sum$

Karlsruhe, November 23, 2010

Student Nr.:

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Worksheet No.5
Advanced Mathematics I

Exercise 21: Prove that there is no inverse function of $f : x \mapsto (x - 1/3)^2$ on $\mathbb{R}$. Show that $f : \mathbb{Z} \to f(\mathbb{Z})$ is invertible. Hint: Apply the equivalence $(x - \frac{1}{3})^2 = (y - \frac{1}{3})^2 \;\Leftrightarrow\; |x - \frac{1}{3}| = |y - \frac{1}{3}|$ and analyse the solution of the second equation.

Exercise 22: Given are the functions f and g with suitable real domains by

$$\text{(i)} \quad f(x) = x^2 - 14x + 45, \qquad \text{(ii)} \quad g(x) = \frac{1}{\sqrt{2-x}} + 5.$$

(a) Determine the maximal domain of these functions.

(b) Investigate if the functions f and g are monotonic and specify their range.

(c) Determine the sets D_f and D_g respectively, such that f is invertible on D_f and g is invertible on D_g. Calculate the inverse functions, their domains and the ranges.

Exercise 23: Consider the piecewise defined function

$$f(x) = \begin{cases} 12 & x < -1 \\ p(x) & -1 \le x < 2 \\ 1 - 2x & x \ge 2\,, \end{cases}$$

where p is a polynomial.

(a) Determine a polynomial p with the smallest possible degree, such that f is continuous. Is it unique?

(b) Can this polynomial be determined uniquely, so that additionally $f(1) = -2$ holds? Justify your answer in case it cannot, or specify it otherwise.

Exercise 24: Let the polynomial $f(x) = x^4 + 5x^3 - 8x^2 + 1 - (x - 2)^3$ be given.

(a) Give the expansion of f around the expansion points $x_1 = 1$ and $x_2 = -1$.

(b) Represent f as a product of linear polynomials. What can you conclude about the behaviour of f in the intervals $[1, \infty)$ and $(-\infty, -3]$?

(c) Sketch the inverse functions of f restricted to $[1, \infty)$ and $(-\infty, -3]$, respectively, and give the appropriate domains and ranges.

Exercise 25: Let $p(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0$ be a polynomial with real coefficients a_k, $k = 0, \ldots, n-1$ and $z \in \mathbb{C}$. Prove that:

(a) If $p(z) = 0$ then $p(\overline{z}) = 0$.

(b) The product of all zeros of the polynomial p is real-valued.

(c) The sum of all zeros of p is real.

Due date: Please hand in your homework until Thursday, December 2, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 5
Advanced Mathematics 1

Exercise T13: Determine the domain D, the range $f(D)$ and the inverse function f^{-1} of

$$f : \begin{cases} D \longrightarrow \mathbb{R}, \\ x \mapsto 1 - \dfrac{1}{x} \end{cases} .$$

Draw a sketch of f and f^{-1}.

Exercise T14: The function f is defined over $\mathbb{R} \setminus \{2\}$ by

$$f(x) = \begin{cases} \dfrac{12x - 9}{x^2 - 2x} & 1 < |x| < 3,\ x \neq 2 \\ p(x) & \text{otherwise} \end{cases},$$

where p is a polynomial. The polynomial p with smallest possible degree should be determined, so that the function f is continuous. Make an assumption for p and prove that this assumption leads to a unique solution. Determine the polynomial.

Exercise T15: Let the function $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \frac{1}{8}\,x^3 + \frac{3}{8}\,x^2 - \frac{9}{8}\,x + \frac{5}{8}$.

(a) Give the expansion of f around the expansion points $x_1 = 1$ and $x_2 = 3$, as well. What can you conclude about the behaviour of f in the intervals $[-5, \infty)$ and $(-\infty, 3]$?

(b) Using a sketch find intervals on which f has an inverse function. Sketch the inverse function.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, November 24, 2010, 14:00-15:30.