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Karlsruhe, December 7, 2010

Student Nr.:

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Worksheet No.7
Advanced Mathematics I

Exercise 31: Examine the following series for convergence.

(a) $\left(\sum\limits_{n=0}^{\infty}\left(\frac{3+4i}{6}\right)^n\right)$, (c) $\left(\sum\limits_{n=1}^{\infty}\frac{2^n n!}{n^n}\right)$,

(b) $\left(\sum\limits_{n=1}^{\infty}\frac{n^2(n+1)^2}{n!}\right)$, (d) $\left(\sum\limits_{n=8}^{\infty}\frac{n+7\sqrt{n}}{n^3-n}\right)$.

Exercise 32: Investigate the convergence and the absolute convergence of the following series:

(a) $\left(\sum\limits_{k=1}^{\infty}(-1)^k\frac{k}{k^2+2k+1}\right)$ (b) $\left(\sum\limits_{k=1}^{\infty}\left[\frac{(-1)^k}{k+3}-\frac{\cos(k\pi)}{k+2}\right]\right)$.

Give for the series in (a) a number $N \in \mathbb{N}$ so that every partial sum s_n for $n \geq N$ differs at most $\frac{1}{8}$ from the limit s.

Exercise 33: Show that the series $\left(\sum\limits_{k=1}^{\infty}\frac{4(k+1)}{k^2(k+2)^2}\right)$ is convergent. Determine also two numbers c_1, c_2 that satisfy $\frac{4(k+1)}{k^2(k+2)^2} = \frac{c_1}{k^2} + \frac{c_2}{(k+2)^2}$ for all $k \in \mathbb{N}$. Use this equation for calculating the series limit s.

Exercise 34: Show that the series $\left(\sum\limits_{k=0}^{\infty}(-1)^k\frac{k+1}{2^k}\right)$ converges absolutely. Next, prove the following representation of its partial sums s_n

$$s_n = \frac{1}{9}\left(4+(-1)^n\frac{3n+5}{2^n}\right), \quad n = 0, 1, 2, \dots,$$

using mathematical induction and use the result for determination of the limit s of this series.

Exercise 35: A parquet recliner has to parquet the whole plane with hexagons. He proceeds step-by-step as in figure below.

(a) Give the explicit representation of the number S_n of the hexagons and the number K_n of the edges in n-th step.

(b) Determine the ratio of the number of the hexagons to the number of the edges if the plane is parqueted completely.

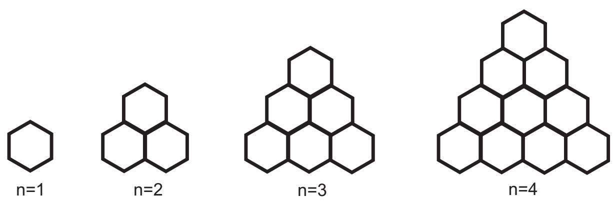

Figure: step-by-step parquetry

Due date: Please hand in your homework until Thursday, December 16, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 7
Advanced Mathematics 1

Exercise T19: Analyze the convergence of the following series using the ratio test, the root test, and the comparison test:

(a) $\left(\sum_{k=1}^{\infty} \frac{1}{k(k+1)}\right)$, (b) $\left(\sum_{k=1}^{\infty} \frac{1}{k^k}\right)$, (c) $\left(\sum_{k=0}^{\infty} \frac{k}{2k+1}\right)$.

Exercise T20: For each of the following series give a general expression for the n-th partial sum, and investigate whether the series is convergent or divergent:

(a) $\left(\sum_{k=0}^{\infty} \left(\frac{2}{2+3i}\right)^k\right)$, (b) $\left(\sum_{k=0}^{\infty} \left(2\sqrt{k} - 4\sqrt{k+1} + 2\sqrt{k+2}\right)\right)$, (c) $\left(\sum_{k=3}^{\infty} \frac{8k}{(k^2-4)^2}\right)$.

Exercise T21: Consider the series $\left(\sum_{n=0}^{\infty} a_n\right)$, where

$$a_n = \begin{cases} \frac{-1}{2^n}, & n \text{ even}, \\ \frac{1}{4^n}, & n \text{ odd}. \end{cases}$$

(a) Show the absolute convergence of the series using the comparison test.

(b) Verify that the root test shows the series to be absolutely convergent, while the ratio test is inconclusive.

(c) Determine the precise value of the series by choosing a suitable decomposition.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, December 8, 2010, 14:00-15:30.