Karlsruhe Institute of Technology
Institute for Algebra and Geometry
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36	37	38	39	40	$\sum$

Karlsruhe, December 14, 2010

Student Nr.:

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Worksheet No.8
Advanced Mathematics I

Exercise 36: Determine the radius of convergence of the power series:

$$\text{(a)}\quad \left(\sum_{k=0}^{\infty} \frac{z^k}{3(k+2)!}\right), \qquad \text{(b)}\quad \left(\sum_{k=1}^{\infty} \frac{z^{2k}\cdot 2^k}{(1+\frac{1}{k})^k}\right), \qquad \text{(c)}\quad \left(\sum_{k=0}^{\infty} k^k z^k\right).$$

Exercise 37: For which $x \in \mathbb{R}$ does the power series

$$\sum_{n=1}^{\infty} (-2)^n \frac{n^2+2}{n^3+n} x^{3n}$$

converge?

Attention: The solution set is an interval. Does it contain its boundary points?

Exercise 38: For $z \in \mathbb{C}$ we set

$$f(z) = 2z \cdot \exp(z^2), \qquad g(z) = \frac{162 - 32z^8}{27 - 18z^2}, \qquad h(z) = f(z) + g(z) - 5 - z - \tfrac{1}{3}z^7.$$

(a) Write these functions as power series $\sum_{k=0}^{\infty} a_k z^k$. Calculate in each case the first nine coefficients $a_0, \dots, a_8$.

Hint: $g(z)$ can be written as a geometric sum.

(b) Mark the points $\left(a_k,\ k - \frac{1}{2}(-1)^k + \frac{1}{2}\right)^\top \in \mathbb{R}^2$ for $k \in \{0, \dots, 8\}$ (only for the function h) in a diagram, connect them and mirror your picture on the y-axis.

Exercise 39: Determine the limit

$$\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{\exp(x^4) - 1}$$

by substituting the respective power series for cos and exp into the given expression.

Exercise 40: Santa Claus, suffering minor rheumatism, is fed up with all the snow. As an early retirement is out of the question he moves from the north pole to the Maledive Islands and decides to deliver his presents henceforth only to places where the total quantity of snow in the coming years is bounded. His personal weather forecast predicts a snow accumulation in the year $2000 + n$ proportional to

$$a_n(x) := \frac{2^n x^{2n}}{(1+\frac{1}{n})^n},$$

where $x \in \mathbb{R}$ is the distance (in 10,000 km) of a given place from the equator.

(a) Calculate the radius of convergence of the power series $\left(\sum_{n=0}^{\infty} a_n(x)\right)$.

(b) Will Santa Claus continue to deliver presents to Karlsruhe?

Hint: The latitude of Karlsruhe is 49°, the circumference of the earth is 40,000 km.

Due date: Please hand in your homework until Thursday, December 23, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

Tutorial 8
Advanced Mathematics 1

Exercise T22: Determine the radius of convergence of the following power series:

(a) $\left(\sum_{k=0}^{\infty} \frac{k+2}{2^k} x^k\right)$, (b) $\left(\sum_{k=1}^{\infty} \frac{(2+x)^{2k}}{(2+\frac{1}{k})^k}\right)$, (c) $\left(\sum_{k=0}^{\infty} \frac{3^{k+2}}{2^k} x^k\right)$.

Exercise T23:

(a) Compute the power series of the rational function $f : \mathbb{C} \setminus \{1\} \to \mathbb{C}$, where $f(z) = \frac{1+z^2}{1-z}$ at the center of expansion $z_0 = 0$.

(b) Compute the radius of convergence of the series.

(c) For which $z \in \mathbb{C}$ does the power series converge?

Exercise T24: Determine the limit

$$\lim_{x \to 0} \frac{x^3}{\sin x - x}$$

by substituting in the power series for sin.

For detailed information regarding this course please check the page
`www.math.kit.edu/iag1/edu/am12010w/en`

Tutorial date: Wednesday, December 15, 2010, 14:00-15:30.