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Student Nr.:

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## Worksheet No.9 Advanced Mathematics I

**Exercise 41:** The following series representation of the logarithm function can be applied to evaluate it approximately on a computer:

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}.$$

- (a) For which  $x \in \mathbb{R}$  is this expansion in a power series possible, i.e. for which  $x$  does the series converge?
- (b) How many elements of the power series are sufficient to evaluate  $\ln(1.5)$  with an accuracy of two decimal places? Calculate  $\ln(1.5)$  with an accuracy of two decimal places by applying the series representation above!

**Exercise 42:**

- (a) Show applying the addition theorem that for  $\varphi \in \mathbb{R}$  and  $n = 0, 1, 2, 3, \dots$  the following equality holds

$$\cos((n+1)\varphi) + \cos((n-1)\varphi) = 2 \cos \varphi \cos(n\varphi).$$

- (b) Consider for  $n = 0, 1, 2, \dots$  the function  $T_n : [-1, 1] \rightarrow \mathbb{R}$  given by

$$T_n(x) := \cos(n \arccos(x)).$$

Prove that

$$T_{n+1}(x) + T_{n-1}(x) = 2x T_n(x).$$

- (c) Conclude that the function  $T_n$  is a polynomial of degree  $n$  at most.
- (d) The function  $T_5$  can be expanded at the expansion point  $x_0 = -1/2$  in a power series. Which radius of convergence does this power series have?



**Exercise 43:** Find all solutions  $z \in \mathbb{C}$  of the equation

$$\cosh z - \frac{1}{2}(1-8i)e^{-z} = 2+2i.$$

Solve the quadratic equation by completing the square.

**Exercise 44:** For each of the following equations, determine the set of solutions  $z \in \mathbb{C}$ :

$$(a) \quad \cos \bar{z} = \overline{\cos z} \qquad (b) \quad e^{i\bar{z}} = \overline{e^{iz}}$$

**Exercise 45:** Solve the following equation for  $z \in \mathbb{C}$ :

$$-5 \cos z + 7i \sin z = 1.$$

**We wish you a merry Christmas and a happy new year!**

**Due date:** Please hand in your homework until Thursday, January 13, 12:00 into the AM1/2-box near seminar room Z1, building 01.85 (Fritz-Erler-Str. 1-3).

## Tutorial 9

### Advanced Mathematics 1

**Exercise T25:**

- (a) Determine all  $z \in \mathbb{C}$ , which satisfy the equation

$$\cos z = 4.$$

Use the exponential representation of the cosine function.

- (b) Determine all complex numbers  $z \in \mathbb{C}$  that satisfy the equation

$$\cosh(z) = -1.$$

Use the representation of cosh in terms of the exponential function.

**Exercise T26:** Prove the following formulas for  $z = x + iy$ ,  $x, y \in \mathbb{R}$ :

$$\begin{aligned}\sin z &= \sin x \cosh y + i \cos x \sinh y, \\ \cos z &= \cos x \cosh y - i \sin x \sinh y.\end{aligned}$$

*Hint:* Use Theorem 4.21.

**Exercise T27:** The power  $a^x$  is defined by  $a^x := e^{x \cdot \ln a}$  for  $a > 0$ ,  $x \in \mathbb{R}$ . Show that:

- (a)  $(a^x)^y = a^{xy}$  for  $a, b > 0$ .  
(b)  $a^x$  is strictly monotonically increasing for  $a > 1$ , and strictly monotonically decreasing for  $0 < a < 1$ .

When  $a = 10$  the inverse function of  $f(x) = 10^x$  is  $\log_{10} x$ .

- (c) How can the value  $\log_{10} x$  be computed, using the function  $\ln x$ ?  
(d) Show that 
$$\begin{aligned}\log_{10}(xy) &= \log_{10} x + \log_{10} y & , \quad x, y > 0 , \\ \log_{10}(x^y) &= y \log_{10} x & , \quad x > 0, y \in \mathbb{R} .\end{aligned}$$

For detailed information regarding this course please check the page  
[www.math.kit.edu/iag1/edu/am12010w/en](http://www.math.kit.edu/iag1/edu/am12010w/en)