Universität Karlsruhe (TH)
Mathematisches Institut II
PDoz. Dr. F.Hettlich
Dipl.-Math.techn. S.Ritterbusch
Dr. H.Schon
Prof. Dr. M.Girardi

46	47	48	49	50	$\sum$

Karlsruhe, June 20, 2005

Student No.: .....

Student No.: .....

Worksheet 10
Advanced Mathematics II for Mechanical Engineering

Problem 46: Find the Laplace transforms of the following functions:

a) $f(t) = 5e^{2t} - 6$,

b) $h(t) = e^{-6t}\sin(2t)$,

c) $g(t) = \begin{cases} \cos(\omega t - \varphi), & \text{für } \omega t - \varphi \geq 0, \\ 0 & \text{otherwise} \end{cases}$
where $\omega, \varphi > 0$,

d) $u(t) = \int_0^t y^3 dy$.

Problem 47: Find the inverse Laplace transforms $f(t)$, $t \in [0, \infty)$ of the functions:

a) $F(s) = \dfrac{2s}{s^4 + 2s^3 + 2s^2 + 2s + 1}$, b) $F(s) = \operatorname{arccot}(s-1)$, c) $F(s) = \dfrac{e^{-\pi s}}{\sqrt{s^2+1}}$.

Hint: a) use partial fraction decomposition; b) and c) try with the table of Laplace transforms.

Aufgabe 48: Using Laplace transform, solve the initial value problem

$$y'''(t) - y''(t) + y'(t) - y(t) = e^{2t}, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = -1.$$

Aufgabe 49: Using Laplace transform, solve the following initial value problem:

$$y''(x) + 2y'(x) + y(x) = -11\cos(2x) + 2\sin(2x), \quad y(0) = 1.$$

Problem 50: Solve the initial value problem for the following system of linear differential equations using Laplace transform:

$$\left.\begin{array}{lll} \dot{x}(t) = x(t) & , & x(0) = 0 \quad , \\ \dot{y}(t) = 2x(t) + y(t) - 2z(t) & , & y(0) = 0 \quad , \\ \dot{z}(t) = 3x(t) + 2y(t) + z(t) + e^t \cos 2t & , & z(0) = 1 \quad , \end{array}\right\} \quad t \in [0, \infty) \quad ,$$

Hint: The dot over the function (e.g. $\dot{x}(t)$) signifies the first derivative with respect to t.

Due date: Monday, June 27, 2005, 1:00 pm (in the slots outside room 208.1 of the Mathematics Building)