

51	52	53	54	55	$\Sigma$

**Worksheet 11**  
**Advanced Mathematics II for Mechanical Engineering**

**Problem 51:** Using the Laplace transform, determine the general solution of the system

$$\begin{aligned} 3y'(t) &= -y(t) + 2z(t) + \sin t + 3 \cos t, \\ 3z'(t) &= 4y(t) + z(t) - 4 \sin t, \end{aligned}$$

**Problem 52:** Using the convolution theorem, determine the original function:

$$\text{a) } \frac{2}{(s+4)(s-1)} \quad \text{b) } \left( 2\frac{e^{-3s}}{s} - \frac{e^{-s}}{s} \right) \frac{s}{s^2+4}$$

**Problem 53:** Determine the solution  $f(x)$  of the Volterra's integral equation

$$f(x) - 2 \int_0^x \cos(x-y)f(y)dy = e^x$$

using Laplace transform.

Hint: Use the convolution theorem.

**Problem 54:**

Let  $f : [0, \infty) \rightarrow \mathbb{R}$  be a 2-periodic function over  $[0, 2]$  defined as:

$$f(t) = \begin{cases} 1 & \text{for } 0 \leq t < 1 \\ 0 & \text{for } 1 \leq t < 2. \end{cases}$$

a) Sketch the function over the interval  $[0, 10]$ .

b) Compute the Laplace transform of  $f$ .

**Problem 55:**

a) Show the norm properties of the Taxicab norm  $\|\cdot\|_1$  and the Maximum norm  $\|\cdot\|_\infty$  in  $\mathbb{R}^n$ .

b) Does  $\|x\| := |x_2|$ ,  $x = (x_1, x_2)^\top \in \mathbb{R}^2$ , define a norm in  $\mathbb{R}^2$ ?

**Due date:** Montag, July 04, 2005, 1:00 pm (in the slots outside room 208.1 of the Mathematics Building)