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Worksheet 12
Advanced Mathematics II for Mechanical Engineering

Problem 56: Given are the vectors $a, b, c \in \mathbb{R}^3$. Show that

- a.) $a \times b = -(b \times a)$ b.) $a \times (b + c) + b \times (a + c) + c \times (a + b) = 0$
 c.) $(a \times b) \cdot c = a \cdot (b \times c) = \det([a, b, c])$ d.) $(a + b) \times (a - b) = 2(b \times a)$

Problem 57: Compute the determinant

$$D = \begin{vmatrix} -3 & 0 & 0 & 7 \\ 6 & 4 & -1 & -3 \\ 0 & -5 & 2 & 2 \\ 3 & -7 & 1 & 0 \end{vmatrix}$$

- a.) by expanding it along the first row,
 b.) by expanding it along the last column,
 c.) using the Gauss algorithm.

Problem 58: Compute the following determinants:

$$\begin{array}{ll} \text{a)} \begin{vmatrix} \cos \varphi & -r \sin \varphi & 0 \\ \sin \varphi & r \cos \varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} & \text{b)} \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{vmatrix} \\ \text{c)} \begin{vmatrix} \cos \vartheta \cos \varphi & -r \sin \vartheta \cos \varphi & -r \cos \vartheta \sin \varphi \\ \cos \vartheta \sin \varphi & -r \sin \vartheta \sin \varphi & r \cos \vartheta \cos \varphi \\ \sin \vartheta & r \cos \vartheta & 0 \end{vmatrix} & \text{d)} \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \end{array}$$

Problem 59: Find the values of $\alpha \in \mathbb{R}$, for which the determinant of

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ -1 & \alpha & 1 & -1 \\ 0 & 0 & 2 & \alpha \\ 1 & 1 & 0 & \alpha + 1 \end{pmatrix}$$

equals 0.

Problem 60: Let's define the matrix $A \in \mathbb{R}^{n \times n}$ by

$$a_{ij} = \begin{cases} x & \text{if } i = j \\ 1 & \text{otherwise} \end{cases}$$

for a parameter $x \in \mathbb{R}$ and a fixed $n \in \mathbb{N}$.

- a) Write out the matrix A for $n = 5$ explicitly.
 b) The determinant $\det(A)$ is a polynomial of x . Give a general formula for it for an arbitrary $n \in \mathbb{N}$.

Due Date: Monday, July 11, 2005, 1:00 pm (in the slots outside room 208.1 of the Mathematics Building)