Universität Karlsruhe (TH)
Mathematisches Institut II
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Karlsruhe, April 18, 2005

Student No.: .....

Worksheet 2
Advanced Mathematics II for Mechanical Engineering

Problem 6: Determine using integration by parts:

a) $\int x^2 \sin x dx$, b) $\int \arctan \frac{1}{x-1} dx$, c) $\int (\ln y)^2 dy$, d) $\int x^3 e^x dx$

Problem 7: Compute using the given substitution:

a) $\int_a \frac{(1+x)^2}{\sqrt{x}} dx, \quad u = \sqrt{x}$, b) $\int \frac{(2+t^2)t^3}{(1+t^2)^3} dt, \quad y = 1+t^2$.

c) $\int_0 a^r dr, \quad a > 1, \quad w = r \ln a$, d) $\int \frac{(1+\sqrt{w})^3}{\sqrt{w}} dw, \quad v = 1+\sqrt{w}$.

Problem 8: Compute the following definite integrals:

a) $\int_{-5}^{-3} x^{2n+1} dx, \quad n \in \mathbb{N}_{\geq 0}$ b) $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$ c) $\int_0^1 3^{\sqrt{2x+1}} dx$ d) $\int_1^2 \frac{(\ln x)^3}{x^2} dx$

Problem 9: Prove the following recursion formula for all $n \in \mathbb{N}$:

$$\int_0^x \sin^n u \, du = -\tfrac{1}{n} \sin^{n-1} x \, \cos x + \tfrac{n-1}{n} \int_0^x \sin^{n-2} u \, du,$$

Using mathematical induction and the formula above, show that

$$\int_0^{\pi/2} \sin^{2m} u \, du = \frac{2m-1}{2m} \cdot \frac{2m-3}{2m-2} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

as well as

$$\int_0^{\pi/2} \sin^{2m+1} u \, du = \frac{2m}{2m+1} \cdot \frac{2m-2}{2m-1} \cdot \ldots \cdot \frac{2}{3},$$

for all $m \in \mathbb{N}$

Problem 10: Determine the general solution of the following differential equations:

a) $y' = \frac{1}{1+x^2}$ b) $y' = x^2 y$ c) $y' = \frac{x}{y^2\sqrt{1+x^2}}$.

Due date: Monday, April 25, 2005, 1:00 pm (in the slots outside room 208.1 of the mathematics building)